# ABSTRACT

Title of Dissertation: ESSAYS ON INNOVATION, FIRM DYNAMICS, AND PRODUCTIVITY GROWTH

Yi Zhao
Doctor of Philosophy, 2020

Dissertation Directed by: Professor John Haltiwanger
Department of Economics

The United States has been experiencing a secular decline in the pace of business formation and young firm activity shares in recent decades. U.S. productivity growth also slowed down during the same period. This thesis studies two questions. First, what are the driving forces and long-term growth implications of the observed trends? Second, how is the creative destruction process translated into the measured cost of living?

Using a longitudinal worker-firm matched dataset from the U.S. Census Bureau, I document that in the innovation intensive high-tech sector, the decline in young firm activity shares is accompanied by: 1) a decline in the growth rate of the demand for skills, and 2) a flattening of the life cycle of skilled labor accumulation of high-tech firms. By developing an innovation-based firm dynamics model that is consistent with the micro-level skilled labor accumulation over the firm life cycle, I show that rising frictions in skilled labor adjustment can explain the joint evolution

of young firm employment shares and demand for skills. These frictions influence productivity growth through affecting the stock of human capital firms possess. A calibrated model shows that a rise in skilled labor adjustment costs lowers productivity growth by 75 basis points in the high-tech sector. A rise in entry costs, on the other hand, is not likely the main driver for declining young firm activities, as it implies an increase in demand for skills. Finally, productivity gain (loss) from reallocation can be offset by the general equilibrium effects of reallocation on aggregate demand for skills.

The impact of innovation on welfare depends critically on taking into account the impact of innovation on the cost of living. Building upon the framework of Redding and Weinstein (2020), I estimate the exact cost of living in the U.S. consumer goods sector using the Nielsen Retail Scanner data over the period of 2006 to 2015. The estimated inflation rate considering product turnover and taste shocks is one percent lower than the tradition CPI measure. Furthermore, I show that the direction of the bias in traditional price indices is determined by the correlation between the initial period market share of products and relative taste shocks. Finally, the exact price index based on a nested CES demand structure can be used to study the contribution to the cost of living by firms of difference sizes.

# ESSAYS ON INNOVATION, FIRM DYNAMICS, AND PRODUCTIVITY GROWTH

by

Yi Zhao

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in fulfillment
of the requirements for the degree of
Doctor of Philosophy
2020

Advisory Committee:
Professor John Haltiwanger, Chair
Professor Boragan Aruoba
Professor John Shea
Professor Felipe Saffie

# Acknowledgments

My greatest debt is to my advisor John Haltiwanger, for his continued support during my entire PhD journey. The interactions with him have always inspired my thinking and guided me to the forefront of the academic research. I am also very thankful to the invaluable opportunities he gave me to work on extremely interesting and challenging projects. It has always been an enjoyable experience working with him and what I have learned from him is way beyond just economics.

I owe my deep gratitude to my committee members Borağan Aruoba, John Shea, and Felipe Saffie. Borağan showed me the state of the art and the rigor of computational macro in early years of my PhD. The macro reading group he organized broadened my horizon and provided an environment for graduate students to practice academic discussion from which I benefited a lot. I am so fortunate to have John reading through every draft of my work and providing detailed comments and edits. The insights from him helped to push my projects forward. Felipe has always been accessible whenever there is a need. Without him, I wouldn't have been able to find my way through the endogenous growth literature.

Macro faculty at the University of Maryland have given feedback and useful comments to my work. I want to thank each one of them: Pierre De Leo, Thomas Drechsel, Nuno Limão, and Luminita Stevens.

I also thank Liu Yang for being the Dean's Representative at my dissertation defense. Liu directed the Federal Statistical Research Data Center at the University of Maryland where all the empirical work in Chapter 2 was done.

I want to thank my fellow graduate students Joonkyu Choi and Seth Murray for in-depth discussions on U.S. Census datasets.

I am grateful to Emin Dinlersoz from the U.S. Census Bureau for his comments on an earlier draft of Chapter 2.

The work in Chapter 3 benefited significantly from discussions with Matthew Shapiro, Gabriel Ehrlich and David Johnson at the University of Michigan. I would like to express my gratitude to them all.

I would also like to express my thanks to Vickie Fletcher for her incredible help that made my graduate study smooth.

Last but not least, I owe my deepest thanks to my family - my mother, father, and husband - who have always stood by me and shared my happiness as well as struggle in this long journey. Without the love, support, and sacrifice from them, I wouldn't have gone this far.

# Table of Contents

# List of Tables

# List of Figures

# Chapter 1: Introduction

Creative destruction, the process that "*incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one*" (Schumpeter (1942)), has been regarded as the essential force underlying economic growth in market economies. In past decades, however, we observe trends that suggest a slowdown of this creative destruction process and a slowdown in productivity growth in the United States. This thesis looks at two issues at the core of these observed trends: first, what are the driving forces behind the observed trends and the implications for long-term growth; and second, how is the creative destruction process translated into the measured cost of living. Firms, the agents which make innovation and investment decisions, are at the center of my analysis.

In Chapter 2, I study possible connections between business dynamism and productivity growth. Young firm activity shares, measured as the employment share of young firms, have been declining in the U.S. and the post-2000 decline has been particularly pronounced in the high-tech sector. Is such decline concerning and does it imply a slowdown in long-term growth in the U.S.? Recent literature hasn't reached a consensus. On the one hand, labor supply side explanations argue that declining young firm activities reflects an efficient response to broader trends such as slower population growth or skill-biased technological progress which lower firm entry rates. On the other hand, some studies argue that frictions play a role and imply inefficiency in the economy.

To tackle this question, I look at an important but neglected angle in the literature - the human capital accumulation of firms. I document, using a longitudinal worker-firm matched dataset (LEHD) from the U.S. Census Bureau, that the post-2000 decline in young firm activities in the high-tech sector has been accompanied by a decline in the growth rate of *demand for skills*. Furthermore, I document that the aggregate decline in the growth of demand for skills in the high-tech sector is driven by a decline in the speed and level of firms' skilled labor's accumulation over their *life cycle*. As skill is the key input to innovation, changes in firms' demand for skills can affect the innovation process and hence productivity growth.

I then develop an innovation-based firm dynamics model that is consistent with micro-level evidence on firms' skilled labor accumulation. Using this model, I show that rising frictions in firms' skilled labor adjustment can be a major driver behind declining young firm activities in the high-tech sector. Moreover, rising adjustment costs could lead to a decline in productivity growth of 75 basis points in the high-tech sector.

Adjustment frictions affect long-term growth through reducing the *stock* of human capital firms possess in equilibrium. A rise in skilled labor adjustment costs raises the marginal cost of hiring skilled labor for all firms, and leads to a decline in the aggregate demand for skills. Such frictions hurt young firms disproportionately more than mature firms as young firms have a higher incentive to adjust the stock of their skilled labor. Young firm employment shares also decline as a result. Long-term growth is hampered in this cases as a lower stock of human capital (skill) leads to less innovation and hence lower productivity growth.

A rise in entry costs, on the other hand, reduces the entry rate and young firm employment share, but does not necessarily lead to lower productivity growth. This is because incumbents' probability of survival increases facing less threat from entrants. An increase in the expected firm value under higher survival rates incentivizes incumbents to hire more skilled labor, and the aggregate demand for skills increases. In equilibrium, incumbents have a higher stock of skilled labor and that leads to an increase in productivity growth.

In sum, Chapter 2 shows that rising entry costs are unlikely to be the driving force behind the decline in young firms' employment shares as they should be associated with a rise in demand for skills. Rising frictions in hiring skilled labor reconcile both patterns, and are concerning as they imply lower long-term growth. My model also suggests that the productivity gain from reallocating skilled labor from old to young firms is not always guaranteed, as higher equilibrium destruction rate will discourage incumbents from hiring skilled labor. This channel offsets the gain from reallocation.

The outcome of innovations can be 1) the creation of new products and the destruction of obsolete ones, 2) the creation and destruction of firms, 3) the improvement of quality of existing products, and 4) changes in product prices due to changes in firms' cost structure. All these will have direct impact on the cost of living and consumer welfare. So how in practice can we capture these innovation outcomes in measured price indices?

Even though in the past, a large amount of research has been done to understand the role of creative destruction on long-term growth, but the work to empirically quantify the impact of innovation on the cost of living and consumer welfare is very limited. This gap is partly due to the lack of product-level price and quantity data at a national scale, as well as the lack of a theoretical framework which incorporates not only product turnover, but also time-varying demand (taste) shocks at product level. In Chapter 3, I study the measurement of the exact cost of living under product and firm turnover and relative taste (demand) shocks, and study the contribution to the cost of living of firms of different sizes.

I utilize the Unified Price Index (UPI) framework proposed by Redding and Weinstein (2020) and transaction-level price and quantity data at the national scale from the Nielsen Retail Scanner Dataset (RMS) to measure the evolution of the exact cost of living in the consumer goods sector in the United States. Derived from a CES demand for a representative consumer, the UPI incorporates product turnover and time-varying demand (taste) shocks to measured price indices. I further construct a UPI under a nested CES demand system where there are firm entry and exit in additional to product entry and exit, and the substitutability of goods within and between firms is allowed to be different.

The estimated UPI suggests that there is quality improvement in consumer products that can not be captured by traditional prices. The average annual inflation in the consumer goods sector measured by the nested UPI is -1.0% on average from 2006 to 2015, compared to 0.03% given by the Laspeyres index. The measured UPI inflation rates are 2.0% and -3.1% for the food and non-food sector respectively, compared to 3.0% and 0.0% from the Laspeyres index.

A key contribution of Redding and Weinstein (2020) is that they argue with the existence of relative taste shocks, Sato-Vartia index is upward biased since increases in tastes are weighted more than reductions in tastes. Such effect is equivalent to an increase in the dispersion of market shares and hence will result in a negative consumer valuation bias term in the UPI. I argue, instead, that the bias is not always positive and the direction of the bias depends on the correlation between initial period market shares and relative taste shocks. I first show this analytically in a two-good economy and then test this result using the Nielsen data. The empirical results support my argument.

Finally, using the nested UPI, I look at the contribution to the cost of living by firms of different sizes. Large firms drive down the cost of living in the aggregate economy by driving down the consumer valuation adjustment. Without the top 5 firms in each product group, the cost of living would have increased by 1.4% on an annual basis. Meanwhile, the role of large firms is different across product groups. In the innovation intensive sector (eg. electronics), large firms have engaged in more active product creation and destruction which drives down the cost of living. On the other hand, the dispersion of market shares within large firms have decreased over time which gives rise to a positive consumer valuation adjustment. In the less innovation intensive sector (eg. snacks), the difference between large and small firms is not significant.

# Chapter 2: Innovation, Demand for Skills, and Productivity Growth

## 2.1 Introduction

[1]The U.S. economy has been experiencing a secular decline in the pace of business formation and young firm activity shares in recent decades. In particular, the post-2000 decline has been very pronounced in the high-tech sector.[2] Should we be concerned about these trends?

Recent literature hasn't reached a consensus on what caused the decline. On the one hand, labor supply side explanations argue that declining young firm activity reflects an efficient response to broader trends such as slower population growth, which leads to lower firm entry rates (Hopenhayn et al. (2018), Karahan et al. (2019)), or skill-biased technological progress, which raises the attractiveness of becoming a worker relative to being an entrepreneur (Salgado (2019)). On the other hand, some studies argue that frictions may affect young firm activity (Davis

[1]Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed. The research in this paper is conducted while the author is Special Sworn Status researcher of the US Census Bureau. This research uses data from the Census Bureau's Longitudinal Employer Household Dynamics Program, which was partially supported by National Science Foundation Grants SES-9978093, SES-0339191 and ITR-0427889; National Institute on Aging Grant AG018854; and grants from the Alfred P. Sloan Foundation.

[2]Literature documents the secular trends include Davis et al. (2012), Hyatt and Spletzer (2013), Decker et al. (2014a), Decker et al. (2014b), Haltiwanger et al. (2014a), Karahan et al. (2019), among others.

and Haltiwanger (2014), Decker et al. (2018), Akcigit and Ates (2019)). Despite a growing literature studying these empirical trends and possible drivers behind declining dynamism, we still don't fully understand the underlying factors, or the impact of declining dynamism on long-term growth.

This chapter studies the possible connections between business dynamism and productivity growth. I focus on demand side factors that affect firms' decision to enter and grow. Using a longitudinal worker-firm matched dataset built from administrative databases from the U.S. Census Bureau, I document a novel fact about the high-tech sector: the post-2000 decline in young firm activity has been accompanied by a decline in the growth rate of *demand for skills*. Furthermore, I document that the aggregate decline in the growth of demand for skills is driven by a decline in the speed and level of firms' skilled labor's accumulation over their *life cycle*.

Motivated by these empirical facts, I develop an innovation-based firm dynamics model that is consistent with micro-level evidence on firms' skilled labor accumulation. Using this model, I study the joint evolution of young firm employment shares and demand for skills in the high-tech sector. My quantitative exercises show that rising adjustment costs in firms' skilled labor adjustment drive down both the young firm employment share and firms' demand for skills, as observed in the data. Moreover, rising adjustment costs lead to a decline in productivity growth of 75 basis points. An increase in entry costs, on the other hand, is not likely to be the dominant driver for the decline in the young firm employment share, as it predicts a counterfactual increase in the demand for skills.

The contribution to the literature is twofold. First, empirically, this chapter is the first to examine demand for skills in the high-tech sector and to document the post-2000 slowdown in its growth. It is also the first to study firms' life cycle of skilled labor accumulation and how this has changed over time. Second, theoretically, I develop an innovation-based endogenous growth model that is consistent with firms' skilled labor accumulation over their life cycle, and use this model to study the joint evolution of the young firm employment share and demand for skills and the implication of rising adjustment costs for productivity growth.

I estimate demand for skills based on the canonical framework developed by Katz and Murphy (1992) and later analyzed in Katz et al. (1999), Autor et al. (2008), and Acemoglu and Autor (2011). The dataset I use is the Longitudinal Employer and Household Dynamics dataset (LEHD), augmented by the Longitudinal Business Database (LBD) and the American Community Survey (ACS) from the U.S. Census Bureau. This dataset covers the universe of U.S. private sector jobs and hence provides us an unbiased view of the economy with a large enough sample size even if we zoom into a narrowly defined sector. I construct composition-adjusted relative wages and relative quantities of skilled to unskilled workers. Assuming the two skill groups are imperfect substitutes, a shift in the demand curve for skills can be inferred from the relative price and quantity series. I find that in the high-tech sector, the growth in the demand for skills slowed down significantly post-2000. The timing coincides with the decline in young firm activity shares in the high-tech sector.

The unique marriage between worker and firm characteristics in LEHD allows me to study not only aggregate demand for skills, but also underlying patterns of firms' skilled labor accumulation. I document how the ratio of the stock of skilled labor to that of unskilled labor changes as firms age in the high-tech sector. I show that firms accumulate skill rapidly when they are young, but the pace of accumulation slows as firms age. I also find that the shape of this life cycle pattern has changed over time. Compared to firms that entered the economy before 2000, firms that were born after 2000 accumulate skilled labor more slowly and tend to have a lower stock of skilled labor when they mature. We refer to this below as a flattening of the life cycle pattern.

The empirical findings motivate me to develop a firm dynamics model with skilled labor accumulation to study the joint evolution of young firm employment shares and demand for skills. This model builds on the endogenous technological change literature and in particular the framework developed by Klette and Kortum (2004), as it delivers a general equilibrium model of technological change while capturing firm entry and exit, and hence is well suited for the analysis of firm dynamics and productivity growth. My model is closely related to the model of Acemoglu et al. (2018), which considers differences between skilled and unskilled labor. I extend those models by introducing adjustment costs to firms when changing their stock of skilled labor. This feature is key to generating a life cycle pattern in the model as we observe in the micro data.

In my model, firms hire skilled labor to perform R&D. A successful innovation provides a technological advantage that enables a firm to take over a competitor's product line. Adjusting the current stock of skilled labor is costly due to the presence of adjustment costs, and this implies that a firm's stock of skilled labor increases as it ages. After a successful innovation, a firm's markup depends on its stock of skilled labor. This assumption captures the fact that it takes time for a young firm to establish a customer base and gain market share. I prove that under general conditions there exists a solution to the model in which a firm's stock of skilled labor per product line increases over firm age and converges to a unique long-run level.

I calibrate the model to be consistent with key features of the high-tech sector for the 1990-2000 period and study how factors that depress young firms' employment share could also affect demand for skills and productivity growth in the high-tech sector. I find that rising frictions in skilled labor adjustment, which raise the marginal cost of hiring skilled labor for all firms, lead to a decline in aggregate demand for skills. Such frictions hurt young firms disproportionately, as young firms have a higher incentive to adjust the stock of their skilled labor. Young firm employment shares also decline as a result. Long-term growth is hampered in this cases, as a lower stock of human capital (skill) leads to less innovation and hence lower productivity growth. An increase in skilled labor adjustment costs that is sufficient to generate the decline in the young firm employment share observed in the data post-2000 leads to a reduction in the long-term productivity growth of 75 basis points.

Rising costs of entry, on the other hand, reduce the entry rate and young firm employment share, but do not necessarily lead to lower productivity growth. This is because incumbents' probability of survival increases when they face less threat from entrants. An increase in the expected firm value due to higher survival rates incentivizes incumbents to hire more skilled labor, and aggregate demand for skills increases. In equilibrium, incumbents have a higher stock of skilled labor, and that leads to an increase in productivity growth. The quantitative impact of reallocation from young to old firms on productivity growth is therefore ambiguous in this case, depending on the relative strength of the two competing forces in equilibrium: the loss from the relative innovation capacity of young vs. old firms, and the gain from an increase in the expected future value of old firms.

In sum, the quantitative study suggests that rising entry costs are unlikely to be driving the decline in young firms' employment shares, as they should be associated with a rising demand for skills. Rising frictions in hiring skilled labor reconcile both patterns, and are concerning as they imply lower long-term growth.

This chapter is connected to several strands of literature. First, it contributes to the literature that studies declining business dynamism, its drivers and implications. Many papers have documented declining entrepreneurship and young firm activities along with declining labor market fluidity in the United States (Davis et al. (2012), Hyatt and Spletzer (2013), Decker et al. (2014a), Decker et al. (2014b), Davis and Haltiwanger (2014), Decker et al. (2018), Molloy et al. (2016), etc.). This chapter complements these empirical studies by documenting a companion feature - declining growth in demand for skills - in the high-tech sector.

Existing studies that develop theoretical frameworks to understand the causes of declining business dynamism cannot also explain this new fact about declining growth in demand for skills. The labor supply story of slower population growth (Hopenhayn et al. (2018) and Karahan et al. (2019)) cannot explain why the share of skilled labor in firms has declined relative to unskilled labor.[3] The skill-based technological progress argument from Salgado (2019) is also unable to reconcile this empirical fact. Salgado (2019) focuses on the entire economy, but if his mechanism - entrepreneurship declines in response to a rising skill premium - holds broadly, we should expect to observe an increase (or at least a slower decline) in the share of young firms in the high-tech sector post-2000, as data suggest that the growth rate of the skill premium fell during that period.

This chapter instead highlights the role of frictions. Davis and Haltiwanger (2014) argue that rising regulations such as occupational licensing or employment protection decrease labor market fluidity and may hurt startups and young firms more. Decker et al. (2018) document weaker marginal responsiveness of businesses to productivity shocks and rising within-industry dispersion of TFP and output per worker in the post-2000 period, consistent with an increase in adjustment frictions. Akcigit and Ates (2019) highlight the decline in knowledge diffusion from frontier firms to laggard firms as the dominant driver underlying declining business dynamism. My work complements this line of literature by highlighting that frictions in adjusting skilled labor could lead to the decline in both young firm activities and demand for skills as observed in the data.

Second, this chapter connects to the literature that studies firm dynamics and productivity. The seminal work of Hopenhayn and Rogerson (1993) shows that an increase in frictions on labor adjustment reduces productivity as it reduces allocative

[3] The argument from Hopenhayn et al. (2018) that the non-production to production worker ratio declined as a result of the aging firm distribution could potentially shed light on the decline in the share of skilled labor, if we assume that non-production workers are mostly skilled labor.

efficiency of factor inputs. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) also study the impact of allocative efficiency on aggregate productivity. The influential work by Hsieh and Klenow (2009) quantifies the role of allocative efficiency for aggregate productivity using firm-level data. They show that distortions that drive wedges between the marginal products of labor and capital across firms will lower aggregate TFP. While they emphasize the role of *allocation* of factor inputs, I focus on the role of the *stock* of factor inputs. Moreover, they focus on the *level* of productivity, while I instead study the *growth* of productivity through an endogenous growth model.

Third, this chapter connects to the endogenous growth literature. The inspiration to look at the impact of the stock of human capital on growth comes from the seminal work of Romer (1990), in which the stock of human capital determines the rate of growth. Romer (1990) builds on a Solow-type model with technological change where human capital affects growth through the nonrivalry property of ideas. I instead model human capital as a direct input to R&D in a firm dynamics model. This model is closely related to the endogenous growth firm dynamics literature (Grossman and Helpman (1991), Aghion and Howitt (1992), Klette and Kortum (2004)) and in particular builds upon Acemoglu et al. (2018). The difference between my model and this line of literature is twofold. First, my model focuses on the age dimension of firms, and in particular is the first to match micro-level life cycle patterns of firms' human capital accumulation. Second, Acemoglu et al. (2018) explore the positive impact on growth from the reallocation of skill from old to young firms, which are assumed to be more innovative. My model, on the other hand, suggests that such a gain from reallocation is not always guaranteed, since in equilibrium reallocation of skilled labor from old to young firms increases the destruction rate, which lowers the expected value of firms. With lower expected future value, firms hire less skilled labor, which leads to lower growth. In other words, I consider how structural changes affect the stock of human capital, how this affects growth, and how this channel offsets the composition effects from reallocation.

Finally, this chapter connects to the literature that studies the evolution of aggregate demand for skills. Autor and Price (2013), Beaudry et al. (2016) and Valletta (2018) document a reversal in the demand for skill and cognitive tasks in the post-2000 period for the U.S. economy. While these studies look at the overall U.S. economy and the labor market impacts of the decline in demand for skills, I focus on the high-tech sector and study the impact of declining demand for skills on the slowdown in productivity growth. I also study a potential cause of this decline in skill demand: a rise in skilled labor adjustment costs.

The rest of the chapter is organized as follows. Section 2 documents the aggregate demand for skills in the high-tech sector and the underlying firms' life cycle of skilled labor accumulation. Section 3 presents the model. Section 4 presents the quantitative results and the last section concludes.

## 2.2 Empirical Evidence

In this section, I document changes over time in the demand for skills and life cycle patterns of skilled labor accumulation for high-tech firms in the United States.

### 2.2.1 Data

The main dataset used for the analysis is the Longitudinal Employer-Household Dynamic (LEHD) dataset from the U.S. Census Bureau. LEHD is a matched employer-employee dataset that covers 95% of U.S. private sector jobs.[4] I use the 2014 snapshot of the data, which covers information from 1990 to 2014. LEHD tracks individual earnings at a quarterly frequency and provides information on worker demographics (e.g. age, gender, education). The unique combination of worker and firm characteristics in LEHD allows me to analyze firm-level behavior regarding skilled labor accumulation and demand for skills.

I augment LEHD with the Longitudinal Business Database (LBD). LBD is a census of business establishments and firms with paid employees in the U.S. and is comprised of survey and administrative records. The LBD tracks business activity information on an annual basis. Data include industry, location, employment, annual payroll, birth, death and ownership changes (if any) at the establishment level.[5] In my analysis, an accurate measure of firm age is important. The fact that LBD provides information on establishments whose identifiers are longitudinally stable as opposed to firm identifiers that can change over time due to ownership, single/multi-unit status or other changes, allows me to have a more robust measure of firm age than the direct usage of firm identifiers from the LEHD, and this is crucial for my analysis. The highest level of business unit ID in the LEHD is the federal EIN. To merge LBD to LEHD, I integrate the federal EIN from the Business Register with

[4]Detailed discussion of LEHD data can be found in Abowd et al. (2009).
[5]Jarmin and Miranda (2002) provides a detailed description of the data.

the LBD and use the crosswalk developed by Haltiwanger et al. (2014c).

Finally, I augment the merged LEHD-LBD data with the American Community Survey (ACS). The education variable in LEHD is heavily imputed, with about 92% of individuals having imputed education (Vilhuber et al. (2018)). To get a sample with a better measure of education, I integrate the demographic information from ACS with the LEHD. After the merge, I keep only the subsample with non-imputed education, which is around 25% of the full sample. As the non-imputed subsample consists of households also appearing in the Decennial Census or ACS, which are random samples of the households in the United States, the 25% non-imputed sample I focus on is representative of the underlying overall sample.

The focus of this chapter is on the high-tech sector. I use the methodology developed by Heckler (2005) and define the high-tech sector as a group of industries with very high shares of workers in the STEM occupations of science, technology, engineering, and math. This sector includes 14 four-digit NAICS industries and covers ICT and biotechnology industries.[6] A list of high-tech industries is provided in Appendix A.1.

The final dataset consists of over $600,000$ firms and over 2 million workers in the high-tech sector from 1990 to 2014.

### 2.2.2 Demand for Skills in the High-Tech Sector

I measure demand for skills following the canonical methodology developed by Katz and Murphy (1992) and further analyzed by Katz et al. (1999), Autor

[6] Similar definitions have been adopted in Haltiwanger et al. (2014b), and Decker et al. (2018).

et al. (2008) and Acemoglu and Autor (2011). The canonical model provides a parsimonious framework for thinking about the skill premium. The key assumption underlying the canonical model is that skilled and unskilled labor are separate inputs for production and are imperfect substitutes. Any force mimicking skilled-biased technological progress can lead to a shift in the demand curve for skills, and the shift can be derived based on how the price and quantity of skilled labor change relative to unskilled labor. In such models, skilled workers are defined as those with college and above education and unskilled workers as those with high school equivalent education.[7] I follow the same definition as the literature.

To calculate demand for skills from the canonical framework, the key is to compute the underlying series for composition-adjusted relative wages and relative quantities between skilled and unskilled workers. The demand for skills can be computed as a linear combination of the relative wage and quantity series given the elasticity of substitution between high and low skilled labor.[8] In the LEHD, I define the wage for a particular worker in year $t$ as the average full quarter earnings for that worker in year $t$ and labor supply as the total number of quarters worked in that year. I classify workers into 64 demographic cells by gender (x2), education (x4) and experience (x8), and compute the average wage and total labor supply for each cell in each year. The average wage for each cell is a weighted average of individuals' quarterly wages where the weights are annual quarters worked. I use

[7] Workers with some college education are spitted into high and low-skill categories evenly.

[8] In particular, I follow Autor et al. (2008) and calculate demand for skills based on the optimality condition $\ln(w_t^H/w_t^L) = (1/\sigma)[D_t - \ln(N_t^H/N_t^L)]$. $w^H$ and $w^L$ are wages for high and low skilled labor respectively and $N^H$ and $N^L$ are corresponding quantities (supply). $D_t$ indexes relative demand shifts favoring high skilled labor. $\sigma$ represents the elasticity of substitution.

fixed weights for each cell to compute the time series of aggregate wages, where the fixed weight for each cell is its average share of labor supply over all years (1990 to 2014). Using fixed weights to aggregate wages across groups has the benefit of keeping the composition of the labor force fixed so that the results are not driven by changes in composition.

I aggregate the sample in every year into 4 cells by education level (college and above, some college, high school and below high school). The relative wage is the wage ratio of college and above workers to high school workers. The relative quantity is the labor supply ratio of college equivalent to high school equivalent workers in efficiency units (where efficiency units of labor supply is defined as the total hours multiplied by the average wage for that cell over the entire sample). The college equivalent labor supply is defined as the sum of labor supply from college and above workers plus half of the supply from some college workers. The high school equivalent supply is the sum of labor supply from high school educated workers, supply from below high school workers and half of the supply from some college workers.[9]

Figure 2.1 plots the evolution of demand for skills in the high-tech sector between 1990 and 2014. The demand for skills was on a steep upward trend from 1990 until 2000, after which the growth in skill demand slowed down significantly. This finding on the high-tech sector resonates with the work of Beaudry et al. (2016) and Valletta (2018), who find a flattening of the wage premium and a reversal of

[9]Note that in the calculation above, effectively, I omit some college and below high school workers from relative wages but include them in relative quantity.

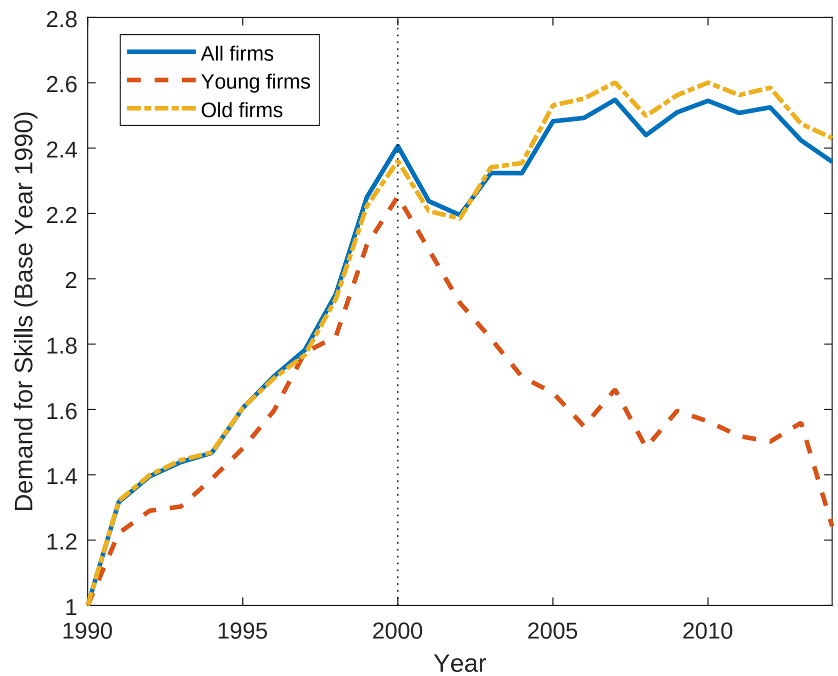


Figure 2.1: Demand for skills by firm age groups in the high-tech sector. The blue solid line shows the demand for skills for the high-tech sector as a whole. The red dashed line shows the demand for skills for young firms (less than 5 years old) and the yellow dash-dot line shows the same measure for old firms. The levels at 1990 are normalized to 1 and the elasticity of substitution between high and low skilled labor is assumed to be 1.62.

the demand for skills and cognitive tasks for the U.S. economy as a whole. Figure 2.1 also breaks down the overall sector demand into the demands of young and old firms, respectively. While the series for old firms tracks the sectoral trend closely, the demand for skills from young firms dropped significantly. Note that the choice of elasticity of substitution is innocuous here. The figure shows the measures assuming an elasticity of 1.62, following Autor et al. (2008), but the significant flattening pattern is robust to elasticities of substitution in the commonly used range of 1 to 3.

The significant slow down of the growth of demand for skills shown in Figure 2.1 is striking as it suggests that some fundamental change regarding skill demand may have occurred in the high-tech sector post-2000. Given the highly innovative nature of this sector and the importance of human capital to innovation and growth, changes in demand for skills may reflect changes in the underlying innovation process that influence productivity growth.

I further examine the relative price and relative quantity components underlying the estimated demand for skills. Panel (A) of Figure 2.2 shows that the skill intensity in young firms has been declining since 2000. This decline in the relative quantity of skilled labor was accompanied by a decline in the relative price of skill in young firms, indicating a drop in skill demand from young firms. Panel (B) of Figure 2.2 shows the same series for old firms. We can see that the skill premium flattened post-2000, reflecting anemic demand for skills.

The definition of skill deserves some discussion. The fundamental measure of the skill of an individual is the amount of human capital they possess. Education has long been used as a proxy for human capital, although some studies have challenged this idea by noting the differences between education and occupation (job task).[10] That line of research argues that human capital is only relevant to the extent it is required by the task a worker is performing. This line of thinking is particularly helpful in analyzing labor market dynamics such as the interaction between technology, offshoring and the structure of wages. However, the difference between education and occupation is arguably less of a concern when we focus on a narrowly

[10]These studies include Autor et al. (2010) and David and Dorn (2013).

(A) Young Firms (B) Old Firms

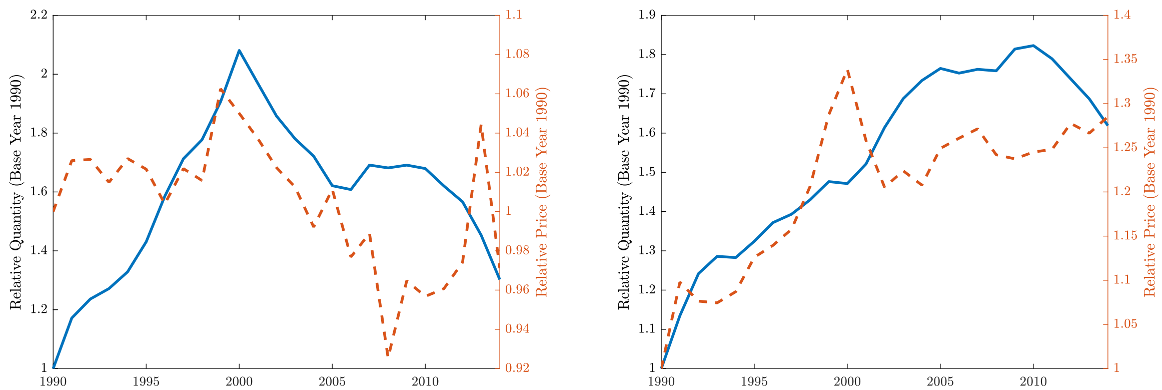


Figure 2.2: Relative quantity and relative wage. Panel (A) shows the evolution of the relative quantity (solid line, LHS) and relative wage (dashed line, RHS) of skilled to unskilled labor in young firms (less than 5 years old). Panel (B) shows the evolution of the relative quantity (solid line, LHS) and relative wage (dashed line, RHS) of skilled to unskilled labor in old firms. The levels in 1990 are normalized to 1.

defined sector in which the relationship between education and tasks is more stable. Moreover, the time span I am focusing on is much shorter compared to the literature emphasizing the importance of tasks. Nonetheless, I provide an occupation (task) based demand for skills measure in Appendix A.2 using publicly available data, and the results are consistent with my baseline results using education to proxy for skills.

The sector-level patterns inspire us to dive deeper into individual firms' skill accumulation and how it has changed over time. The next section discusses the details.

### 2.2.3 Life Cycle of Skill Accumulation among High-Tech Firms

The unique combination of firm and worker information in the LEHD allows me to track how firms accumulate skill over time. Before looking into skill accumulation over the firm's life cycle, it is useful to think again about the measurement of skill.

The literature on skill demand following Katz and Murphy (1992) emphasizes that in order to track aggregate relative quantities and prices for skilled and unskilled labor, one must adjust for changes in the composition of the pool of workers so that the measured quantity of skill is comparable over time. The same intuition holds when we want to track the quantity of skill a firm possesses over time. To make such a composition adjustment, I convert the number of workers to the equivalent "efficiency units" by multiplying the quantity of workers in a given cell by a conversion coefficient that is specific to that cell. Cells are defined based on gender, age, education and experience. The conversion factor is defined in the same way as in Autor et al. (2008), as the average relative wage in that group over the entire period from 1990 to 2014.

The life cycle pattern of skill accumulation is calculated as a firm's skill intensity (efficiency units of high-skilled labor divided by efficiency units of low-skilled labor) of that firm at age $a$ relative to age 0.

Figure 2.3 shows the life cycle pattern of skill intensity of high-tech firms. We observe that high-tech firms accumulate skills rapidly when they are young (less than 5 years old) and that the speed of accumulation then slows down. Firms have a relatively stable skill intensity when they mature.

I further examine how this life cycle pattern has changed over time. To do so, I first split sample firms into cohorts defined by year of entry, and separately plot the life cycles of skilled and unskilled labor relative to age 0 for those cohorts. Figure 2.4 shows the results. Panel (A) of figure 2.4 shows the life cycle of the

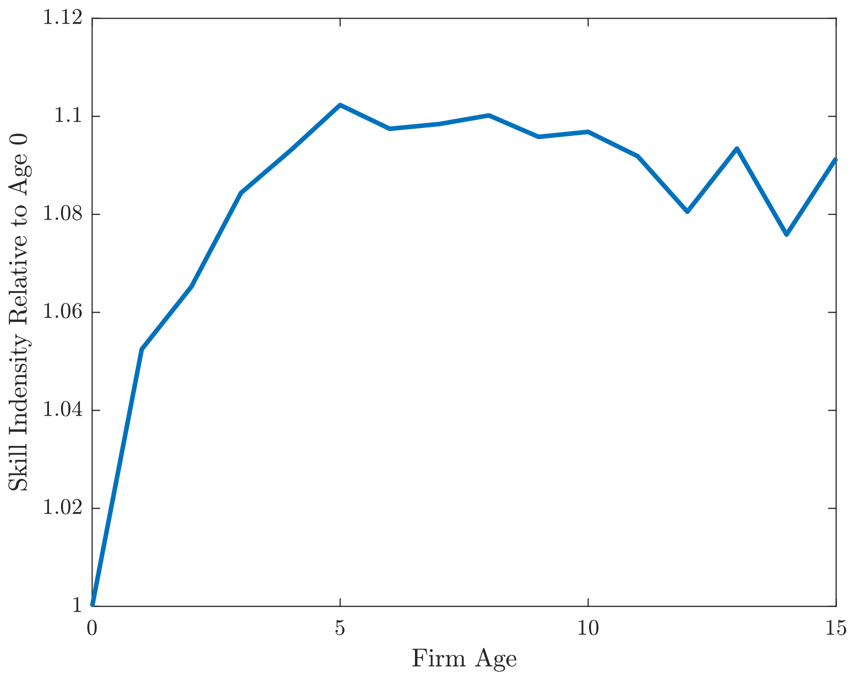


Figure 2.3: The figure plots the average life cycle of skill accumulation for sample high-tech firms. Specifically, the ratio of high skilled labor in efficiency units to low skilled labor in efficiency units at age a relative to age 0. Firms do not need to survive for 15 years in order to be included.

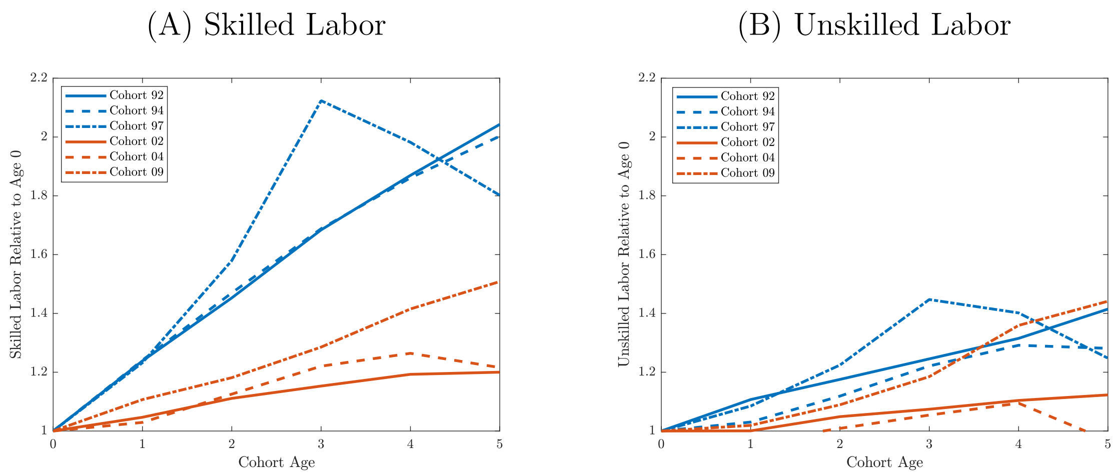


Figure 2.4: Life cycle of skill for different cohorts Panel A shows skilled labor relative to age 0 for cohorts entering the economy at different times. Blue indicates cohorts entering before 2000 and red indicates cohorts entering after 2000. Panel B shows unskilled labor relative to age 0 for cohorts entering the economy at different times. Blue indicates cohorts entering before 2000 and red indicates cohorts entering after 2000.

amount of high-skilled labor relative to age 0 for different cohorts as they age. The difference between cohorts entering into the economy before and after 2000 is significant. Cohorts entering before 2000 almost double their level of skilled labor by age 5, but cohorts entering after 2000 increase their skilled labor by less than 50 percent by age 5. On the other hand, the differences between the life cycle patterns of unskilled labor accumulation are less significant for cohorts entering before and after 2000, as shown in Panel (B) of figure 2.4.

Finally, I run a fixed effects regression to estimate the life cycle pattern of skill, controlling for firm and time fixed effects. Specifically, I run the following regression:

$$\Delta \ln E^{S,U}_{i,a,t} = \alpha_i + \beta_t + \gamma_a + \epsilon_{a,i,t} \tag{2.1}$$

(A) Skilled Labor (B) Unskilled Labor

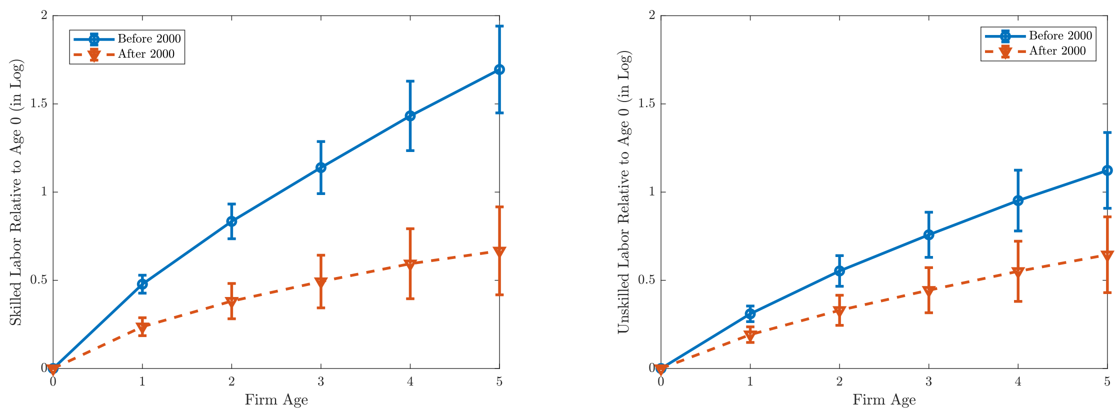


Figure 2.5: Fixed effects estimates. Panel (A) shows the fixed effects estimates of the life cycle pattern of skilled labor accumulation for periods before and after 2000 respectively. Panel (B) shows the fixed effects estimates of the life cycle pattern of unskilled labor accumulation for periods before and after 2000 respectively. Error bars indicate 90% confidence intervals.

for the periods before and after 2000, respectively. $E^{S,U}_{i,a,t}$ are the efficiency units of skilled or unskilled labor for firm $i$ with age $a$ at time $t$. $\alpha_i$, $\beta_t$, and $\gamma_a$ are firm, time and age fixed effects. $\Delta$ is the change relative to age 0.

Figure 2.5 plots the estimates of $\gamma_a$ in equation 2.1, along with 90% confidence intervals. Panel (A) suggests that firms accumulate skilled labor much more rapidly before 2000 than in the later period. The confidence intervals of the two estimates do not overlap, suggesting that differences between life cycle patterns before and after 2000 are statistically significant. Panel (B) plots the life cycle patterns for unskilled labor. There is also a significant flattening of the life cycle patterns for unskilled labor, but the gap is much smaller than that of unskilled labor. Taking Panels (A) and (B) together, we can see that the skilled labor accumulation in high-tech firms slowed down considerably post-2000.

## 2.3 The Model

In this section, I introduce the theoretical framework and characterize the stationary balanced growth equilibrium.

### 2.3.1 Final Good Production

The economy has a representative firm that combines intermediate inputs to produce a final good, according to the following production function:

$$\ln Y_t = \int_{j \in \Omega_t} \ln y_{jt} dj, \tag{2.2}$$

where $y_{jt}$ is the input of intermediate good $j$ at time $t$, and $\Omega_t \in [0,1]$ is the set of active product lines at time $t$. $\mathcal{M}_t$ is the measure of $\Omega_t$ and can be smaller than 1. The reason why there can be inactive product lines will be made clear later. The final good is used for consumption.

For each intermediate good $j$, the final good producer can choose from $N_j$ versions of that good, where the total amount of input $j$ satisfies

$$y_{jt} = \sum_{k=1}^{N_j} x_{jt}^k, \tag{2.3}$$

where $x_{jt}^k$ is the amount of version $k$ of intermediate good $j$ at time $t$. Following the standard assumption in the literature (Grossman and Helpman (1991)), I assume that different versions are perfectly substitutable, which implies that the final good producer will use the version with the lowest price.

The optimization problem of the final good producer implies

$$p_{jt}y_{jt} = \frac{1}{\mathcal{M}_t} P_t Y_t, \tag{2.4}$$

which further implies that

$$\ln P_t = \ln \mathcal{M}_t + \frac{1}{\mathcal{M}_t} \int_{j \in \Omega_t} \ln p_{jt} dj, \tag{2.5}$$

We choose the final good as the numeraire, i.e. $P_t = 1$.

### 2.3.2 Intermediate Good Production

Intermediate good (product) $j$ is produced by the monopolist who has the leading-ledge technology in that product line. A firm can own multiple product lines and produce multiple intermediate goods simultaneously. Firms in the intermediate goods sector hire both skilled and unskilled labor. I assume that after paying a fixed cost of $l^f$ in skilled labor, a firm has access to a linear production technology of the following form:

$$y_{jt} = A_{jt} l^u_{jt}, \tag{2.6}$$

where $l^u_{jt}$ is the number of unskilled workers employed for producing this good, and $A_{jt}$ is the leading-edge technology of firm $f$ on this product line $j$. The marginal cost of production is therefore $w^u_t / A_{jt}$.

I assume that the firm producing version $k$ of intermediate good $j$ has productivity $A_{jt}^k$ and engages in Bertrand monopolistic competition as often assumed in the endogenous technical change literature, where the firm with the lowest marginal cost wins the whole market and sets its price equal to the marginal cost of its closest follower.[11] Assume the productivity of the leading edge firm for good $j$ is $A_{jt} = \max_k A_{jt}^k$ at time $t$, with that of the closest follower being $\hat{A}_{jt} = \max_{l, A_{jt}^k \neq A_{jt}} A_{jt}^k$. Due to Bertrand competition, the equilibrium price of good $j$ is

$$p_{jt} = \frac{1}{\hat{A}_{jt}} w_t^u. \tag{2.7}$$

### 2.3.3 Firm Heterogeneity and the Innovation Process

Firms in the intermediate goods sector engage in both product and process innovation. Product innovations enable firms to acquire new products, and process innovations can further affect the technology advantage of a firm compared to its closest competitors.

Firms improve product quality through process innovation. Denote the quality advantage of the leading edge firm $A_{jt}/\hat{A}_{jt}$ by $q_{jt}$, with $q_{jt} > 1$. I assume that $q_{jt}$ is determined by the amount of skilled labor used in the product line:

$$q_{jt} = q(l_{jt}^s) = \frac{1}{1 - \eta_0 \times (l_{jt}^s)^{\eta_1}}, \tag{2.8}$$

where $\eta_0, \eta_1 \in [0, 1]$. $l^s$ is skilled labor net of the fixed cost $l_f$. Intuitively, if firm

[11] Grossman and Helpman (1991), Lentz and Mortensen (2008), Ates and Saffie (2016), among others.

has no skilled labor, i.e. $l^s_{jt} = 0$, it cannot enjoy a technology edge over its nearest competitor. The size of the quality advantage is an increasing function of skilled labor devoted to that product line.

The introduction of $\eta_1$ deserves some discussion. I assume this specific form so that the uniqueness of the equilibrium can be ensured through certain regularity conditions.

I model product innovations following Klette and Kortum (2004), so that the number of product lines of a firm of age $a$, $n_{a,t}$, changes through a creative-destruction process. The likelihood of success is heterogeneous across firms, depending on the amount of skilled labor per product line of a firm, and the number of product lines owned by the firm. The latter can be considered as a proxy of the knowledge capital of that firm.

Product innovations are undirected. A firm that owns $n_{a,t}$ product lines receives $n_{a,t}$ iid innovation shocks. Each shock follows a Bernoulli distribution with the success probability $\lambda(l^s_{a,t})$, in which $\lambda(\cdot)$ is an increasing and concave function, and $l^s_{a,t}$ denotes the skilled labor per product line. I assume a parsimonious form $\lambda(l^s_{a,t}) = \lambda_0(l^s_{a,t})^\theta$ with $\theta < 1$.

Denote the equilibrium destruction rate in the economy as $\mu_t$. Conditional on the survival of a firm (with respect to exogenous destruction, which will be introduced shortly), a product line with skilled labor $l^s_{a,t}$ faces the following three possible outcomes next period:

$$\text{The number of product lines} = \begin{cases} 2 & \text{with probability } \lambda(l^s_{a,t})(1-\mu_t) \\ 1 & \text{with probability } (1-\lambda(l^s_{a,t}))(1-\mu_t) + \lambda(l^s_{a,t})\mu_t \\ 0 & \text{with probability } (1-\lambda(l^s_{a,t}))\mu_t \end{cases}.$$

The expected number of product lines for the firm is therefore

$$E[n_{a+1,t+1}] = [1 + \lambda(l^s_{a,t}) - \mu_t] \times N_{a,t}. \tag{2.9}$$

Firms need to pay adjustment costs when adjusting their skilled labor. I assume that the amount of skilled labor is the same across product lines owned by a firm, and that the adjustment cost is defined as

$$\phi_t(n_{a,t}, l^s_{a-1,t-1}, l^s_{a,t}, w^s_t) \equiv \frac{\varphi}{2} w^s_t n_{a,t} l^s_{a-1,t-1} \left[\frac{l^s_{a,t} - (1-\delta)l^s_{a-1,t-1}}{l^s_{a-1,t-1}}\right]^2, \tag{2.10}$$

where $\delta$ is an exogenously given separation rate of skilled labor, and $\varphi$ determines the difficulty of adjusting skilled labor. I can alternatively assume the adjustment cost is on the total skilled labor at the firm, but as I will show later, defining adjustment costs based on the number of skilled workers per product line will help simplify the optimization problems of firms.

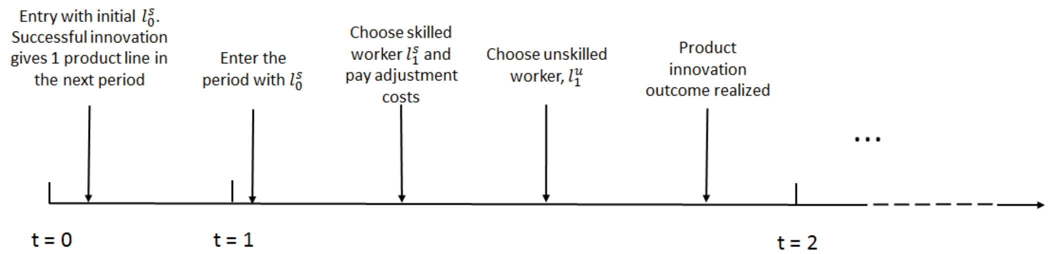


Figure 2.6: The timeline of the model.

### 2.3.4 Optimal Decisions of Intermediate Goods Producers

The timeline of the model is illustrated by figure 2.6. I solve the optimization problem of intermediate goods producers in two steps. In the first step, I solve for the optimal amount of unskilled labor as a function of the amount of skilled labor per product line. In the second step, I solve the optimal choice of the amount of skilled labor per product line.

Consider a product line $j$ owned by a firm of age $a$. Given the amount of skilled labor per product line $l^s_{a,t}$, the firm chooses unskilled labor of the product line $l^u_{jt}$ by maximizing operating profits:

$$B_{a,j,t} = \max_{\{l^u_{jt}\}} \left( p_{jt} y_{jt} - w^u_t l^u_{jt} \right) \tag{2.11}$$

$$\text{s.t. } y_{jt} = A_{jt} l^u_{jt}$$

$$y_{jt} = \frac{Y_t}{M_t p_{jt}}$$

$$p_{jt} = \frac{w^u_t}{\tilde{A}_{jt}}.$$

The firm's optimal choice of unskilled labor is

$$l_{jt}^u = \frac{Y_t}{M_t q_{jt} w_t^u}, \tag{2.12}$$

and this implies the operating profit $B_{a,j,t}$ is given by

$$B_{a,j,t} = \frac{Y_t}{M_t}(1 - \frac{1}{q_{jt}}) = \frac{Y_t}{M_t}\eta_0 (l_{a,t}^s)^{\eta_1}. \tag{2.13}$$

Therefore, all products owned by the firm have the same operating profit $Y_t \eta_0 (l_{a,t}^s)^{\eta_1}$, which is only a function of $l_{a,t}^s$, and which henceforth is denoted as $B_t(l_{a,t}^s)$.

### 2.3.5 Entry and Exit

I assume that firms' entry decisions follow Acemoglu et al. (2017). A new firm chooses to hire $l_0^s$ units of skilled labor and needs to pay a fixed cost to enter into the economy. After paying the fixed cost, the entrant will have access to the innovation technology $\lambda^E(\cdot)$. A successful innovation enables an entrant to own one product line at the beginning of the next period.

I assume that the fixed cost is equal to $\xi$ units of skilled labor, and that the remaining $l_0^s - \xi$ units of skilled labor are involved in the innovation process. The free entry condition gives

$$\max_{l_0^s} \left\{ \lambda^E (l_0^s - \xi) \mathbb{E} \frac{V(l_0^s)}{1+r} - w^s l_0^s \right\} = 0, \tag{2.14}$$

where $\lambda^E(l) = \lambda_0^E l^\theta$.

The number of entrants adjusts so that the implied destruction rate $\mu$ ensures that the free entry condition holds.

The firm faces an exogenous destruction rate of $\nu$. Firms exit the economy when their number of product lines decreases to zero or when they are hit by the exogenous destruction shock.

### 2.3.6 Value Functions

Now I turn to the second step. Adjustment costs of skilled labor imply that the amount of skilled labor per product line is a state variable. Firms choose the optimal level of skilled labor by solving the following Bellman equation:

$$\begin{aligned} V_t(n_{a,t}, l^s_{a-1,t-1}) = \max_{\{l^s_{a,t} > l_f\}} \Big\{ & n_{a,t} B_t(l^s_{a,t}) - n_{a,t} w^s_t l^s_{a,t} - \phi_t(n_{a,t}, l^s_{a-1,t-1}, l^s_{a,t}, w^s_t) \\ & + \frac{1-\nu}{1+r} \mathbb{E}\big[V_{t+1}(n_{a+1,t+1}, l^s_{a,t})\big] \Big\}. \end{aligned} \tag{2.15}$$

The form of adjustment costs implies that the value function is linear in the number of product lines $n_{a,t}$. Define $v_t(l^s_{a-1,t-1}) = V_t(n_{a,t}, l^s_{a-1,t-1})/n_{a,t}$.

**LEMMA 1:** The Bellman equation (2.15) can be simplified to

$$\begin{aligned} v_t(l^s_{a-1,t-1}) = \max_{\{l^s_{a,t} > l_f\}} \Big\{ & B_t(l^s_{a,t}) - w^s_t l^s_{a,t} - \frac{\varphi}{2} w^s_t l^s_{a-1,t-1} \Big[\frac{l^s_{a,t} - (1-\delta) l^s_{a-1,t-1}}{l^s_{a-1,t-1}}\Big]^2 \\ & + (1-\nu) \frac{1 + \lambda(l^s_{a,t}) - \mu_t}{1+r} v_{t+1}(l^s_{a,t}) \Big\}. \end{aligned} \tag{2.16}$$

*Proof*: It is straightforward to prove the linearity of $V$ in $n$ using guess and verify.

### 2.3.7 Household

There is a representative household who maximizes the lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \ln \left[ C_t - A_t \tau (L_t^s)^{\chi} \right], \tag{2.17}$$

where $C_t$ is the consumption of the final good at time $t$. $\chi > 1$ and $\frac{1}{\chi - 1}$ denotes the Frisch elasticity of labor supply. As will be defined later, $A_t$ is the aggregate productivity, where $A_t = Y_t / L_t^u$. Note that $A_t$ changes endogenously in response to innovation. I choose this endogenous scaling by following Ates and Saffie (2016) who justify this assumption through home production. This assumption is useful for simplifying the analysis of the balanced growth path later.

The representative household owns $L^u$ units of unskilled labor, which is constant over time, and supplies $L_t^s$ units of skilled labor, which will be supplied elastically based on the skilled labor wage $w_t^s$.

The budget constraint of the household is:

$$C_t = w_t^s L^s + w_t^u L^u + \Pi_t,$$

where $\Pi_t$ is the aggregate profit, defined as

$$\Pi_t = \int_{j \in \Omega_t} \pi_{jt} dj, \tag{2.18}$$

where

$$\pi_{jt} = B_t(l^s_{jt}) - w^s_t l^s_{jt} - \frac{\phi_t(n_{i,t}, l^s_{i,t-1}, l^s_{i,t}, w^s_t)}{n_{i,t}},$$

where $i$ denotes the firm that owns product line $j$ in period $t$, and $l^s_{i,t}$ denotes the average amount of skilled labor owned by firm $i$ in period $t$.

Aggregate profit as a share of total output is

$$\zeta_t = \frac{\Pi_t}{Y_t} = \frac{1}{Y_t}\int_{j\in\Omega_t} B_t(l^s_{jt}) - w^s_t l^s_{jt} - \frac{\phi_t(n_{i,t}, l^s_{i,t-1}, l^s_{i,t}, w^s_t)}{n_{i,t}} dj.$$

The first order condition of the household optimization problem implies that the the aggregate supply of skilled labor satisfies:

$$w^s_t = A_t \tau \chi (L^s_t)^{\chi-1}. \tag{2.19}$$

### 2.3.8 Aggregate Growth Dynamics

As the final good is the numeraire,

$$\begin{aligned} 0 = \ln P_t &= \ln M_t + \frac{1}{M_t}\int_{j\in\Omega_t} \ln p_{jt} dj \\ &= \ln M_t + \frac{1}{M_t}\int_{j\in\Omega_t} (\ln w^u_t - \ln \tilde{A}_{jt}) dj, \end{aligned}$$

which implies that

$$\ln w^u_t = \frac{1}{M_t}\int_{j\in\Omega_t} \ln \tilde{A}_{jt} dj - \ln(M_t). \tag{2.20}$$

As an individual product line's unskilled labor is given by equation (2.12), I can define the aggregate level of unskilled labor as

$$L_t^u = \frac{Y_t}{M_t w_t^u} \int_{j \in \Omega_t} \frac{1}{q_{jt}} dj, \tag{2.21}$$

which yields

$$\begin{aligned}\ln Y_t &= \ln w_t^u + \ln M_t + \ln L_t^u - \ln\Big(\int_{j \in \Omega_t} \frac{1}{q_{jt}} dj\Big), \\ &= \frac{1}{M_t} \int_{j \in \Omega_t} \ln(\tilde{A}_{jt}) dj + \ln L_t^u - \ln\Big(\int_{j \in \Omega_t} \frac{1}{q_{jt}} dj\Big).\end{aligned}$$

We can define the aggregate productivity

$$A_t = Y_t / L_t^u, \tag{2.22}$$

where $A_t$ satisfies

$$\begin{aligned}\ln A_t =& \frac{1}{M_t} \int_{j \in \Omega_t} \ln \tilde{A}_{jt} dj - \ln\Big(\int_{j \in \Omega_t} \frac{1}{q_{jt}} dj\Big) \\ =& \frac{1}{M_t} \int_{j \in \Omega_t} \ln A_{jt} dj - \frac{1}{M_t} \int_{j \in \Omega_t} \ln q_{jt} dj - \ln\Big(\int_{j \in \Omega_t} \frac{1}{q_{jt}} dj\Big).\end{aligned}$$

### 2.3.9 Equilibrium

Since the amount of skilled labor per product line is the only state variable, it is the same across all firms entering within the same cohort, and therefore is always the same for all firms within the same cohort over time. This implies that $l_{a,t}^s$ and $l_{a,t}^u$ are functions solely of age in period $t$.

Denote the measure of product lines owned by firms of age $a$ at the beginning of period $t$ as $\Lambda_{a,t}$, and the measure of entrants in period $t$ as $\Lambda_{0,t}$.

A *competitive equilibrium* is defined as prices $\{\{p_{jt}\}_{j\in\Omega_t},\ w_t^s,\ w_t^u\}$ and choices $\{Y_t,\ \{y_{jt}^D\}_{j\in\Omega_t},\ \{y_{j,t}^S\}_{j\in\Omega_t},\ \{l_{a,t}^s\}_{a=0,1,\ldots}\}, \{l_{a,t}^u\}_{a=1,\ldots}\},\ C_t,\ L_t^s\}$, profit $\Pi_t$, destruction rate $\mu_t$, and the distribution of products across age cohorts $\{\Lambda_{a,t}\}_{a=0,1,\ldots}$, such that

1. $\{C_t, L_t^s\}$ solve the decision problem of households (2.17) taking $w_t^s, w_t^u$ and $\Pi_t$ as given, in which $\Pi_t$ satisfies equation (2.18).

2. $Y_t$ and $\{y_{jt}^D\}$ solve the representative final goods producer's problem (A.1), taking $\{p_{jt}\}_{j\in\Omega_t}$ as given.

3. $\{y_{jt}^S, l_{a,t}^u\}$ solve the intermediate goods producer of age $a$'s problem (2.12), taking $Y_t, p_{j,t}$, and $l_{a,t}^s$ as given, if the product line $j$ belongs to the cohort aged $a$ in period $t$.

4. $\{l_{a,t}^s\}_{a=0,1,\ldots}$ solve the Bellman equation (2.16), taking $w_t^s$ and $\mu_t$ as given.

5. The free entry condition $\max_{l_{0,t}^s}\left\{\lambda^E(l_{0,t}^s-\xi)\frac{v_t(l_{0,t}^s)}{1+r}-w_t^s l_{0,t}^s\right\}=0$ is satisfied, in which $v_t$ satisfies the Bellman equation (2.16).

6. Market clearing conditions hold:

$$C_t + \sum_{a=1}^{\infty} \Lambda_{a,t} \frac{\phi_t(n_{a,t}, l^s_{a,t-1}, l^s_{a,t}, w^s_t)}{n_{a,t}} = Y_t \tag{2.23}$$

$$y^D_{j,t} = y^S_{jt} \tag{2.24}$$

$$\sum_{a=1}^{\infty} \Lambda_{a,t} l^u_{a,t} = L^u \tag{2.25}$$

$$\sum_{a=0}^{\infty} \Lambda_{a,t} l^s_{a,t} = L^s_t, \tag{2.26}$$

where $L^s_t$ satisfies equation (2.19).

7. The distribution of products across age cohorts evolves in an internally consistent way:

$$\Lambda_{1,t+1} = \Lambda_{0,t} \lambda^E (l^s_{0,t} - \xi) \tag{2.27}$$

$$\Lambda_{a+1,t+1} = \Lambda_{a,t} \times \big(1 + \lambda(l^s_{a,t}) - \mu_t\big)(1 - \nu) \tag{2.28}$$

8. The equilibrium destruction rate $\mu_t$ satisfies:

$$\mu_t = \lambda(l^s_{0,t} - \xi)\Lambda_{0,t} + \sum_{a=1}^{\infty} \lambda(l^s_{a,t})\Lambda_{a,t}, \tag{2.29}$$

where $M_t$ is the share of product lines alive at the beginning of period $t$, and this equation holds because I assume that innovation is undirected.

To simplify the analysis, I add an assumption that due to technological spillovers, the productivity of idle production line is on average the same as that of active product line

**Assumption 1** $\frac{1}{M_t}\int_{j\in\Omega_t}\ln A_{jt}dj = \frac{1}{1-M_t}\int_{j\notin\Omega_t}\ln A_{jt}dj$.

In this chapter, I study the *balanced growth path* or the *steady state* of the equilibrium.

Denote $\tilde{X} = X_t/A_t$ for a variable $X$ that is growing at the same rate as $A_t$, and $X = X_t$ for a variable that is a constant at the steady state. In particular $A_t$: $\tilde{Y} = Y_t/A_t$, $\tilde{w}^s = w^s_t/A_t$, $\tilde{w}^u = w^u_t/A_t$, $\tilde{v} = v_t/A_t$, $\tilde{B}(l^s_a) = B^t(l^s_{a,t})/A_t$. Variables that are constant at the steady state include: $\mu = \mu_t$, $l^s_a = l^s_{a,t}$, $L^s = L^s_t$, $\Lambda_a = \Lambda_{a,t}$,and $M_t = M$, in which $a \in \{0, 1, ..., +\infty\}$.

**LEMMA 2:** Along the balanced growth path,

$$M = \frac{\mu}{1-(1-\mu)(1-\nu)}. \tag{2.30}$$

*Proof:* See Appendix A.6.

The following equations hold in the steady state:

$$\tilde{Y} = L^u$$

$$\tilde{B}_a = L^u\eta_0(l^s_a)^{\eta_1}$$

$$\tilde{w}^s = \tau\chi(L^{s,s})^{\chi-1}$$

$$\tilde{w}^u = \frac{1}{M}\sum_{a=1}^{\infty}\Lambda_a\frac{1}{q(l^s_a)}$$

$$\zeta = \frac{1}{L_u} \sum_{a=1}^{\infty} \Lambda_a \times \left( \tilde{B}_a - \tilde{w}^s l_a^s - \frac{\varphi}{2} \tilde{w}^s l_{a-1}^s \Big[ l_a^s - (1-\delta) l_{a-1}^s l_{a-1}^s \Big]^2 \right)$$

$$\begin{aligned} \tilde{v}(l_{a-1}^s) = \max_{l_a^s} \Big\{ & \tilde{B}_a - \tilde{w}^s l_a^s - \frac{\varphi}{2} \tilde{w}^s l_{a-1}^s \Big[ \frac{l_a^s - (1-\delta) l_{a-1}^s}{l_{a-1}^s} \Big]^2 \\ & + \frac{1-\nu}{1+r} (1 + \lambda(l_a^s) - \mu) \tilde{v}(l_a^s) \Big\} \end{aligned}$$

$$\max_{l_0^s} \Big\{ \lambda^E (l_0^s - \xi) \frac{\tilde{v}(l_0^s)}{1+r} - \tilde{w}^s l_0^s = 0 \Big\}$$

$$\Lambda_1 = \Lambda_0 \lambda^E (l_0^s - \xi),$$

$$\Lambda_{a+1} = \Lambda_a \times (1-\nu) \big( 1 + \lambda(l_a^s) - \mu \big)$$

$$\mu = \lambda^E (l_0^s - \xi) \Lambda_0 + (1-\nu) \times \sum_{a=1}^{\infty} \lambda(l_a^s) \Lambda_a$$

$$L^s = \sum_{a=0}^{\infty} \Lambda_a l_a^s$$

$$L^u = \sum_{a=1}^{\infty} \Lambda_a l_a^u.$$

The steady state growth rate of aggregate productivity is

$$g_A = \ln \frac{A_t}{A_{t-1}} = \Lambda_1 (\ln q(l_0^s)) + \sum_{a=1}^{\infty} \Lambda_a (1-\nu) \lambda(l_a^s) \ln(q(l_a^s)).$$

### 2.3.10 The Optimal Choices of Skilled Labor

I solve the steady state equilibrium in three steps. In the first step, I solve the optimal decisions $\{l_a^s, \tilde{v}(l_a^s)\}$ as functions of $\{\tilde{w}^s, \tilde{w}^u, \mu\}$. In the second step, I determine $\mu$ based on the entry condition. Finally, I use market clearing conditions to solve for equilibrium prices $\tilde{w}^s$ and $\tilde{w}^u$. In this section, I discuss the first step and present conditions under which there exists a unique solution to the skilled labor life cycle accumulation problem.

I conjecture that the number of skilled workers per product line will asymptotically approach a constant level $\bar{l}$ as firm age rises, where $\bar{l}$ satisfies conditions (2.31) (2.32), which are derived by letting $\bar{l} = l_a^s = l_{a+1}^s$ and solving the Bellman equation for a firm aged $a$. (For details, please refer to the derivation of (A.9) in the appendix).

$$\varphi\tilde{w}^s\delta^s = \frac{d\tilde{B}^s(\bar{l})}{d\bar{l}} - \tilde{w}^s + \frac{1+\lambda(\bar{l})-\mu}{1+r}\frac{\varphi\tilde{w}^s}{2}(1-(1-\delta)^2) + \frac{\lambda'(\bar{l})}{1+r}v^s(\bar{l}) \tag{2.31}$$

$$\frac{r+\mu-\lambda(\bar{l})}{1+r}v^s(\bar{l}) = \tilde{B}^s(\bar{l}) - \tilde{w}^s\bar{l} - \frac{\varphi}{2}\tilde{w}^s\bar{l}\delta^2, \tag{2.32}$$

where $\tilde{B}^s(\bar{l}) = L^u\eta_0\bar{l}^{\eta_1}$, and $d\tilde{B}^s(\bar{l})/d\bar{l} = \eta_1 L^u\eta_0(\bar{l})^{\eta_1-1}$.

Plugging (2.32) into (2.31), we have:

$$\varphi\tilde{w}^s\delta - \frac{d\tilde{B}^s(\bar{l})}{d\bar{l}} + \tilde{w}^s - \frac{1+\lambda(\bar{l})-\mu}{1+r}\frac{\varphi\tilde{w}^s}{2}(2\delta-\delta^2) = \frac{\lambda'(\bar{l})}{r-\lambda(\bar{l})+\mu}\Big(\tilde{B}^s(\bar{l}) - \tilde{w}^s\bar{l} - \frac{\varphi}{2}\tilde{w}^s\bar{l}\delta^2\Big). \tag{2.33}$$

Note that the upper bound for $\bar{l}$, $\bar{l}_{max}$, is given by

$$\bar{l} \leq \bar{l}_{max} = \Big(\frac{r+\mu}{\lambda_0}\Big)^{\frac{1}{\theta}}.$$

The intuition for $\bar{l}_{max}$ is as follows: the maximum innovation rate of firms should not exceed $r+\mu$. With the assumption that the maximum innovation rate of firms does not exceed $\mu$, the higher the innovative capacity of a firm, the fewer skilled workers there should be, to avoid violating the condition that the maximum innovation rate should be smaller than $r+\mu$.

The existence and uniqueness of $\bar{l}$ in the economy are guaranteed under the conditions outlined in Proposition 1, the proof of which can be found in appendix (A.5).

**Proposition 1: The existence and uniqueness of $\bar{l}$.** Under the following conditions, there is a unique solution of $\bar{l}$:

$$\eta_1 < \theta \tag{2.34}$$

$$\theta + \eta_1 < 1 \tag{2.35}$$

$$\Big[\frac{B_0}{(1+\frac{\varphi}{2}\delta^2)\tilde{w}^s - l_f}\Big] \times \big(\frac{r+\mu}{\lambda_0}\big)^{\frac{\eta_1 - 1}{\theta}} < 1 \tag{2.36}$$

$$\frac{\delta\varphi(2-\delta)}{2+\varphi\delta^2} < \frac{(1-\eta_1)(1-\theta)(1+r)}{(1+\theta-\eta_1)(r+\mu)} \tag{2.37}$$

*Proof:* See appendix A.5.

In the quantitative analysis below, I will restrict parameters such that these conditions hold. I describe the computation algorithm in Appendix A.7.

## 2.4 Quantitative Analysis

### 2.4.1 Calibration

The calibration strategy consists of two parts. I determine the values of standard parameters outside of the model, based on the standard practice in the literature. Second, I calibrate other parameters internally such that the balanced growth path of the model matches several features of the high-tech sector between 1990 and 2000.

I choose the elasticity of successful innovation with respect to R&D as 0.5, following Acemoglu et al. (2018). The interest rate is set to 0.02, similar to those used by Acemoglu et al. (2018) and Akcigit and Kerr (2018). The Frisch elasticity $\chi$ is set to be 1.455, following Mendoza (1991) and also standard in the literature. The depreciation rate of human capital captures not only depreciation of the knowledge stock of a firm, but also the separation of skilled workers from firms. I select its value to be 0.1, the same as a typical choice in the literature for the depreciation rate of capital. The results are not sensitive to a smaller value of this depreciation rate. Table 2.1 summarizes the parameters calibrated outside of the model.

Other parameters are determined inside the model by matching model outcomes with their data counterparts. For the purpose of our analysis, it is key to match the moments governing skilled labor, including the life cycle of skilled labor accumulation, the payroll share of skilled relative to unskilled labor, and the distribution of skilled labor among different firm age groups. I design the calibration

| **Parameter** | **Symbol** | **Value** | **Sources/Data Targets** |
|---|---|---|---|
| | | *Parameters from Other Studies* | |
| Innovation elasticity | $\theta$ | 0.50 | Acemoglu et al. (2018) |
| Discount rate | $r$ | 0.02 | Acemoglu et al. (2018), Akcigit and Kerr (2018) |
| Frisch elasticity | $\chi$ | 1.455 | Standard |
| Depreciation of human capital | $\delta$ | 0.1 | Standard |

Table 2.1: Parameters Determined Outside the Model

strategy to make sure my model matches those key features of the data.

I determine process innovation capacity parameters $\eta_0$ and $\eta_1$ by matching the average payroll share of skilled relative to unskilled labor and the young firm employment share in skilled labor in the LEHD from 1990 to 2000. These two data targets are sensitive to process innovation, because process innovation affects the skill accumulation over firm age through the marginal return of hiring an additional skilled worker. I define the young firm employment share, in both the model outcome and the data counterpart as the employed share of skilled workers for firms aged 0 to 4. The results are not sensitive to using all workers rather than skilled workers.

The adjustment cost intensity parameter $\varphi$ is chosen to match the life cycle growth of skilled labor in the LEHD for 1990-2000. The life cycle growth of skill, as explained earlier, is defined as the efficiency units of skilled labor at age $a$ relative to that at age 0. I look at the growth from age 0 to age 5 for an average high-tech firm between 1990 and 2000, which in the data is taken from the coefficient of the fixed effects regression in equation (2.1).

The entrant and incumbent innovation capacity $\lambda_E$ and $\lambda_0$, the fixed operation

| Parameter | | Value |
|---|---|---|
| *A. Data target: payroll share of skilled relative to unskilled workers and young firm employment share of skilled labor* | | |
| Scaling of quality improvement | $\eta_0$ | 0.31 |
| Payroll sensitivity to skilled labor | $\eta_1$ | 0.30 |
| *B. Data target: life cycle of skilled labor* | | |
| Skilled labor adjustment cost | $\varphi$ | 1 |
| *C. Data targets: distribution of skilled labor across firm age, entry, and exit rates* | | |
| Incumbent innovation intensity | $\lambda_0$ | 0.01 |
| Entrant innovation intensity | $\lambda_E$ | 0.0074 |
| Fixed cost | $l^f$ | 1.45 |
| Entry cost | $\xi$ | 0.45 |
| Exogenous destruction rate | $\nu$ | 0.005 |

Table 2.2: Parameters Determined Inside the Model

costs $l_f$, the fixed cost of entry $\xi$, and exogenous destruction rate $\nu$ are jointly chosen by matching the skilled labor distribution across firm age groups and firm exit rates computed from the LEHD for 1990-2000.

Table 2.2 summarizes the value of parameters and Table 2.3 shows the model performance.

### 2.4.2 Quantitative Results

To highlight the importance of skilled labor adjustment frictions for long-term productivity growth, I compare the growth impact of two exogenous changes in model parameters. The first is an increase in the adjustment cost of changing the stock of skilled labor of a firm, and the second is a decline in the entrant innovation capacity. The former represents frictions that affect all incumbent firms and the

| Data Target | Data | Model |
|---|---|---|
| *A. Data target: payroll share of skilled relative to unskilled workers and young employment share* | | |
| The ratio of non-production to production worker compensation | 2.9 | 2.9 |
| Share of skilled labor in firms aged 0-4 | 9.6% | 9.8% |
| *B. Data target: life-cycle of skilled labor* | | |
| Age 5 to Age 0 ratio of skill intensity | 5.5 | 5.5 |
| *C. Data targets: distribution of skilled labor across firm age, entry, and exit rates (LEHD 90-14)* | | |
| Share of skilled labor in firms aged 5-7 | 5.4% | 7.2% |
| Share of skilled labor in firms aged 8-10 | 5.4% | 6.7% |
| Share of skilled labor in firms aged 11-15 | 8.9% | 9.9% |
| Exit rate (small/young) | 9.9% | 6.5% |
| Exit rate (small/old) | 8.8% | 6.4% |

Table 2.3: Data Targets and Model Counterparts

latter is a way to capture frictions facing entrants only. I will show how the young firm employment share, the life cycle of skilled labor accumulation, the demand for skills, and productivity growth respond to the two different types of shocks.

I discipline the change in adjustment costs and entry innovation capacity so that the young firm employment share decreases from 9.6% in the pre-2000 period to 6.2% by 2014 along the balanced growth path.[12] I do not target changes in the life cycle of skilled labor accumulation. To match the decline in young firm employment shares, in the first experiment, entrant innovation capacity declines from 0.0074 to 0.0065, other things equal. In the second experiment, the skilled labor adjustment cost parameter increases from 1 to 10 [13], while keeping other parameters unchanged.

[12]These are the young firm employment shares in terms of the skilled labor. Young firms are those less than 5 years old. 9.6% is the 10-year average of the share between 1990 and 2000, and 6.2% is the young firm employment share in 2014.

[13]The share of adjustment costs in total revenue increases from 0.05% to 5 percent

I consider a decline in entrant innovation capacity to proxy for rising entry costs as recent studies suggest that the innovation production function may have changed over time (Bloom et al. (2017), Fernald and Jones (2014)). The results are broadly consistent if I instead study rising fixed costs of entry.

I show that rising frictions in skilled labor adjustment can generate declines in the young firm employment share and the demand for skills, consistent with the data, while declining entry costs cannot generate a decline in the demand for skills. I further show that a rise in skilled labor adjustment costs sufficient to generate a decline in the young firm employment share consistent with the data would imply a 75 basis point decrease in the productivity growth rate in the high-tech sector.

#### 2.4.2.1 Skilled Labor Distribution by Firm Age

The skilled employment distribution across firm age is sensitive to both entrant innovation capacity and skilled labor adjustment costs. Figure 2.7 shows that a decline in young firm innovation capacity reduces the skilled employment shares of firms aged between 0-4, 5-7, 8-10 and 10-15. Similarly, increasing adjustment costs also shift the skilled labor distribution towards old firms.

While both lower entrant innovation capacity and higher adjustment costs shift the distribution of skilled labor towards old firms, they operate through different channels. Decreasing entrant innovation capacity lowers the entry rate from 2.9% to 1.9%. This reduction in the entry rate brings down young firm employment shares. Increasing adjustment costs, on the other hand, has limited impact on the

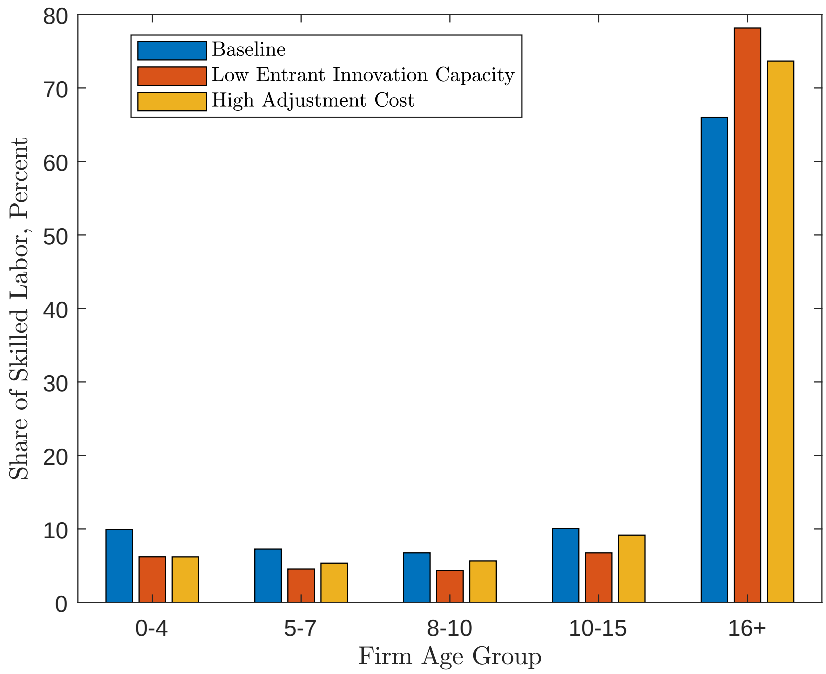


Figure 2.7: Distribution of skilled labor across firm age groups under different scenarios. Blue bars (left) show the baseline distribution. Red bars(center) represent the share of skilled labor owned by a particular firm age group under low entrant innovation capacity. Yellow bars (right) represent the share of skilled labor owned by a particular firm age group under high adjustment costs. Both lower entrant innovation capacity and increased skilled labor adjustment costs shift the skilled labor distribution towards old firms.

entry margin. The entry rate decreases by only 30 basis points, to 2.6%. The main channel through which adjustment costs affect the young firm employment share is the intensive margin. Facing higher adjustment costs, both young and old firms hire less skilled labor, but the reduction is more significant for young firms, as they have a higher incentive to hire skilled labor. The difference between the implications of lower entrant innovation capacity and higher adjustment costs can also be seen from Figure 2.7, where the gap between the base case the low entrant innovation capacity case is similar for firms in age groups 0-4, 5-7, 8-10 and 10-15, while this gap is larger for firms in the younger age groups in the case of high adjustment costs.

#### 2.4.2.2 The Life Cycle of Skilled Labor Accumulation

Declining entrant innovation capacity and rising skilled labor adjustment costs have opposite impacts on the life cycle of skilled labor accumulation, as is shown in Figure 2.8. Declining entrant innovation capacity induces incumbents to hire more skilled labor. The intuition is that the marginal return of hiring an additional skilled worker increases with the expected probability of survival, which increases with a decline in the entrant innovation capacity. The equilibrium destruction rate $\mu$ reduces to 6.1% from 6.8% when I lower the entrant innovation capacity.

Rising adjustment costs, meanwhile, clearly discourage firms from hiring skilled workers. The marginal cost of hiring an additional unit of skilled labor increases for both young and old firms. Even though the destruction rate also decreases in this case (from 6.8% to 6.4%), the marginal benefit of hiring is not sufficiently large

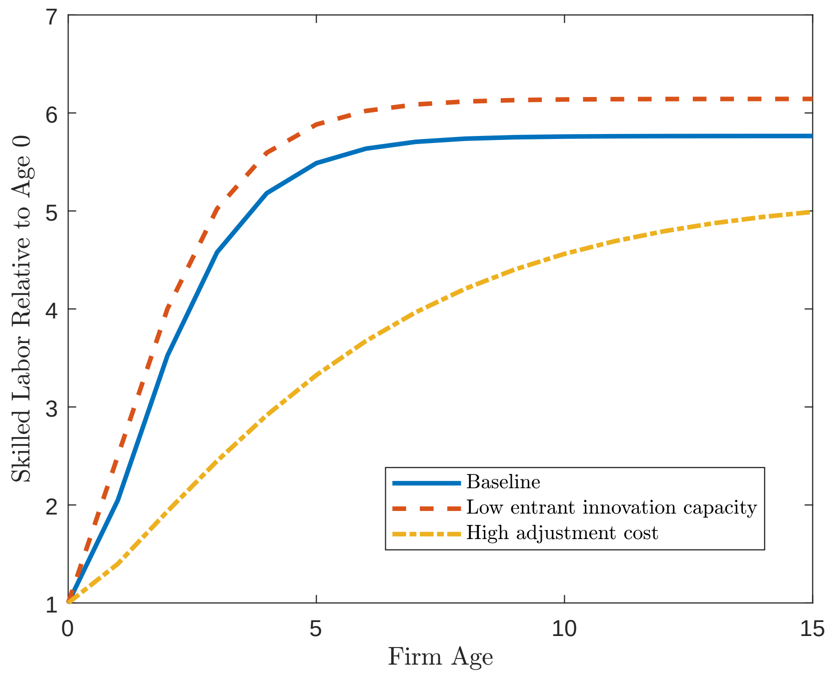


Figure 2.8: This figure plots skilled labor relative to age 0 under different scenarios. The solid blue line is the baseline case. The red dashed line is the life cycle under lower entrant innovation capacity and the yellow dash-dot line is the life cycle under higher skilled labor adjustment costs. Entrant innovation capacity and skilled labor adjustment costs have different impacts on the life cycle of skilled labor accumulation.

to offset the increased costs. Hence, in the equilibrium all firms hire fewer skilled workers and the life cycle of skilled labor accumulation flattens, as shown in Figure 2.8.

One may wonder if the flattened life cycle under higher adjustment costs is simply the result of a higher initial level of skilled labor, as entrants may choose to increase their initial stock of skilled labor to avoid paying higher adjustment costs later on. We show that this effect is minimal. Initial stock of the skilled labor increases only slightly from 2.4 to 2.7, while the stock of skilled labor decreases in much large magnitudes for incumbents of all ages (for example, the stock of skilled labor for an age 5 firm decreases from 18.3 to 10.6).

#### 2.4.2.3 Aggregate Demand for Skills

The aggregate demand for skills responds differently to declining entrant innovation capacity and rising adjustment costs. A decline in entrant innovation capacity leads to a strong rise in the aggregate demand for skills, as measured by the wage ratio between high and low skilled labor, which rises from 3.0 to 3.6. The increase in demand for skills occurs in response to the reduced equilibrium destruction rate, which increases the survival rate and hence the expected value of incumbents. An increase in the skilled labor adjustment costs, on the other hand, leads to a significant decline in the aggregate demand for skills, as the wage ratio drops to 2.4 from 3.0.

I further break down the demand for skills into relative price and quantity,

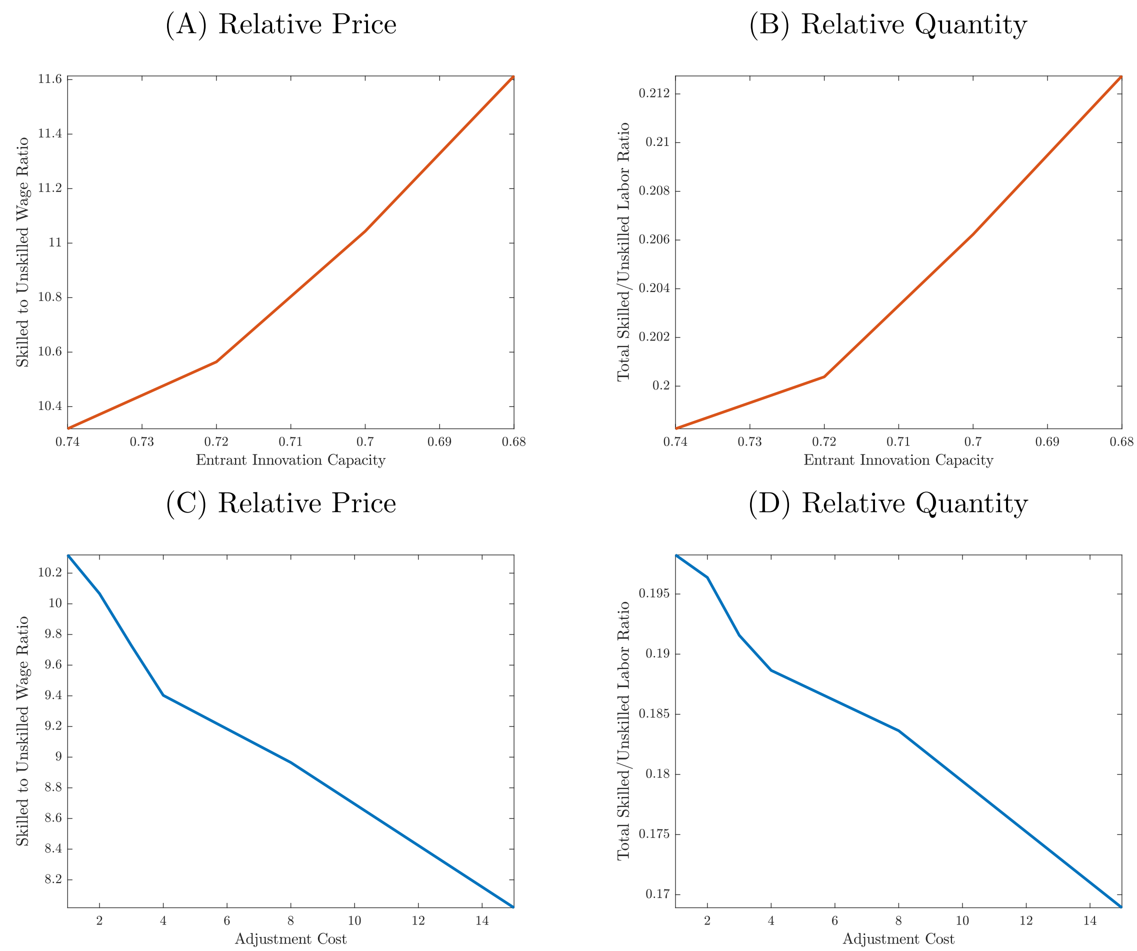


Figure 2.9: Relative price and quantity under different scenarios. Panel (A) shows the high to low skilled labor wage ratio when entrant innovation capacity declines. Panel (B) shows the ratio of aggregate high to low skilled labor when entrant innovation capacity declines. Panel (C) shows the high to low skilled labor wage ratio when skilled labor adjustment costs increase. Panel (D) shows the ratio of aggregate high to low skilled labor when skilled labor adjustment costs increase.

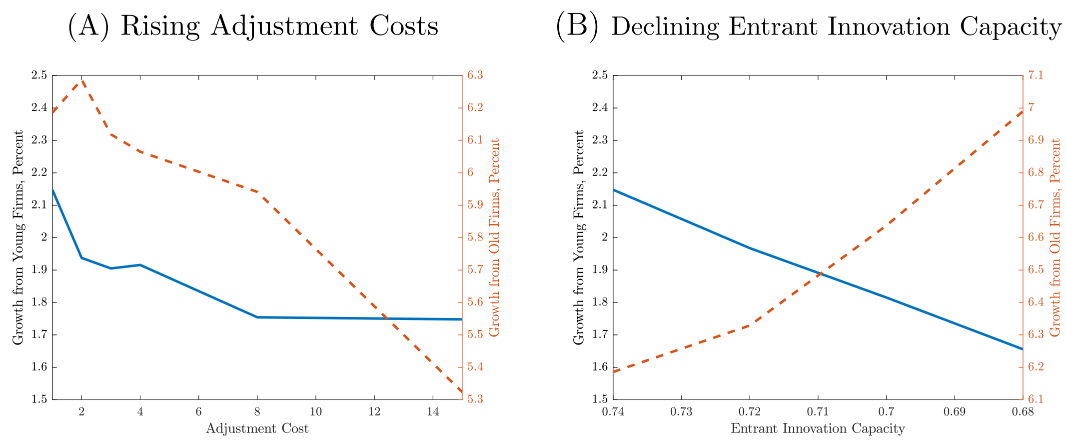


Figure 2.10: Sources of growth. Panel (A) shows the growth from young firms (solid line, LHS) and old firms (dashed line, RHS) under rising skilled labor adjustment costs. Panel (B) shows the growth from young firms (solid line, LHS) and old firms (dashed line,RHS) under decreasing entrant innovation capacity. Young firms are those less than 5 years old.

as shown in Figure 2.9. A decline in entrant innovation capacity increases both the relative price (wage) and the relative quantity of skilled to unskilled labor. An increase in skilled labor adjustment costs, on the other hand, decreases both the relative price and quantity of skilled to unskilled labor.

#### 2.4.2.4 Implications for Productivity Growth

I now look at the implications of changes in entry costs and adjustment costs for productivity growth under the two different scenarios. When skilled labor adjustment costs increase, productivity growth decreases by 75 basis points. On the other hand, when entrant innovation capacity declines, aggregate productivity growth actually increases by 55 basis points. The counterintuitive productivity gain is a result of the increased demand for skills, which increases the stock of skilled labor for incumbent firms and hence produces a higher innovation success rate and higher productivity growth.

The effects can be better seen in Figure 2.10, where I decompose aggregate productivity growth into growth from young firms (including entrants) and growth from old firms. Panel A shows the growth components under rising adjustment costs. We can see that when adjustment costs are higher, both young and old firms' growth rates decline. Panel B shows the growth components under decreasing entrant innovation capacity. When entrant innovation capacity decreases, the growth from entrants declines. But such a decline is offset by the increase in the growth rate of incumbents, whose innovation success rate increases as they now face a lower threat of destruction and hire more skilled labor.

#### 2.4.2.5 Alternative Experiments

The analysis above considers entrant innovation capacity as a proxy for entry costs. Alternatively, I experiment with increasing the fixed cost of entry $\xi$, while fixed costs of entry which leads to broadly similar implications as declining entrant innovation capacity. Figure 2.11 shows the results. Increasing fixed costs of entry lowers the entry rate and decreases the young firm employment share. Meanwhile, the lowered threat from entrants increases the probability of survival of incumbents and their demand for skills. In equilibrium, the increase in productivity growth from incumbents more than offsets the slower growth from entrants, similar to the effects of lower entrant innovation capacity. The difference comes from the implications for life cycle growth. When fixed costs of entry increase, firms need to hire a higher

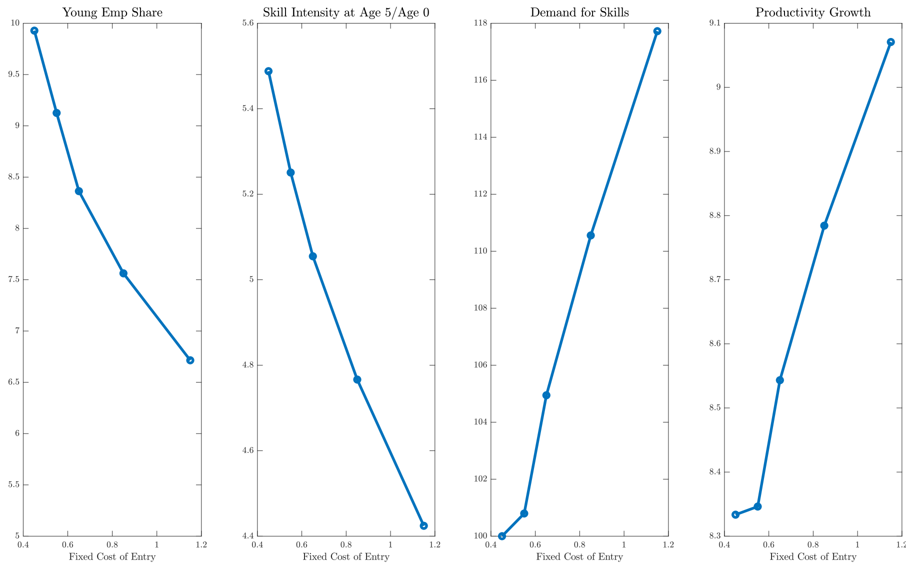


Figure 2.11: Effects of fixed costs of entry. I show how the young firm employment share, the life cycle of skilled labor accumulation, the demand for skills, and productivity growth response to a change in the fixed cost of entry.

amount of skilled labor in order to enter which leads to slower growth of skilled labor post-entry.

I next discuss the effects when lowering the innovation capacity for entrants and incumbents together, while keeping the relative strength of entrant and incumbent innovation capacity constant. Figure 2.12 shows the results. Intuitively, decreasing innovation capacity will lower aggregate growth. In fact, in this case, both the life cycle of skilled labor and the demand for skills remain largely unchanged, while productivity growth from both entrants and incumbents declines purely as a result of reduced innovation capacity. The young firm employment share declines slightly in this case. Intuitively, when innovation capacity declines, both entrants and incumbents have a lower success rate of product innovation. However, incum-

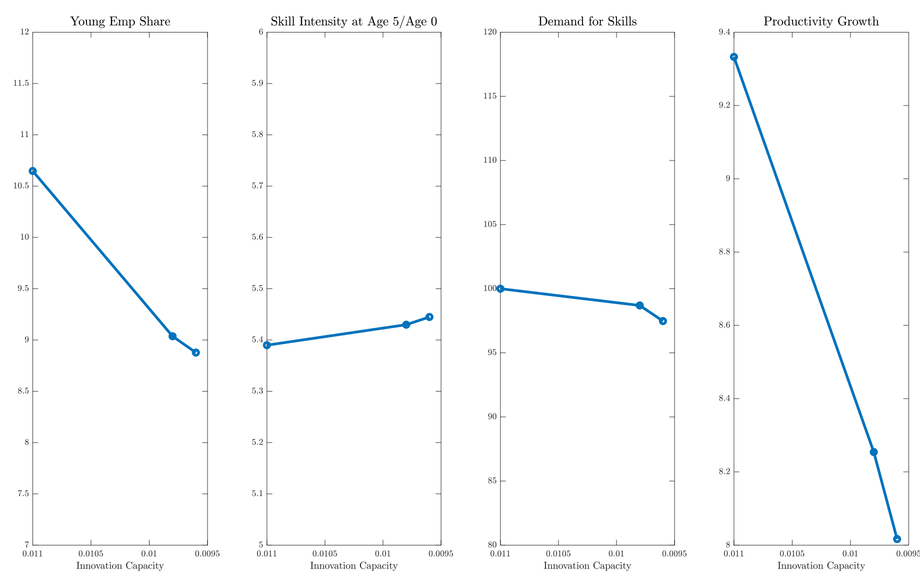


Figure 2.12: Effects of lower innovation capacity for both entrants and incumbents. I show how the young firm employment share, the life cycle of skilled labor accumulation, the demand for skills, and productivity growth response to a change in innovation capacity.

bents' advantage in process innovation now becomes higher, as they have a higher stock of skilled labor to conduct process innovation (as reflected in the quality improvement/mark up function equation 2.8).

Finally, note that in my analysis, the decline in the life cycle pattern of skill accumulation under rising adjustment frictions is not as large as the post-2000 decline from the data. If there are additional factors which further reduce firms' demand for skills, we should expect that the productivity growth will be further hampered.

## 2.5 Concluding Remarks

This chapter studies the drivers behind declining business dynamism and its implications for productivity growth. Using a longitudinal worker-firm matched dataset that covers the universe of U.S. private sector jobs, I document that the decline in young firm activity as accompanied by a decline in the growth of demand for skills in the high-tech sector post-2000. Moreover, I document that the decline in aggregate demand for skills was accompanied by a flattening of the life cycle of skilled labor accumulation among high-tech firms.

I develop an endogenous growth firm dynamics model to study the joint evolution of young firm activity and demand for skills. I incorporate adjustment costs for skilled labor into the standard framework so that the model can be consistent with the micro level data patterns of firms' life cycle of skill accumulation.

I show that rising frictions in skilled labor adjustment can reconcile the empirical patterns observed in the data, i.e. a decline in the young firm activity share, a flattening of the life cycle of firms' skilled labor accumulation and a decline in the aggregate demand for skills. Moreover, aggregate productivity growth is hampered in this case, as firms accumulate a lower stock of skilled labor when they face higher adjustment costs. By calibrating the model to the high-tech sector, I find that rising adjustment frictions could lead to a 75 basis points decrease in productivity growth in the high-tech sector.

The model also suggests that the productivity gain from reallocating skilled labor from old to young firms is not always guaranteed, as a higher equilibrium destruction rate will discourage incumbents from hiring skilled labor. This channel offsets the gain from reallocation. Admittedly, in the current model, young firms are equally innovative as old firms, which may leave interesting post-entry dynamics unexplored. Future research may want to include richer post-entry dynamics to better assess the relative strength of the two offsetting channels in equilibrium.

Finally, even though my focus has been on the high-tech sector, the methodology is by no means restricted to this sector alone. Aggregate patterns suggest that the decline in demand for skills may exist more broadly outside the high-tech sector, and it would be interesting to explore the connection between the demand for skills and declining business dynamism and productivity growth in the broader U.S. economy.

# Chapter 3: Innovation, Firm Size, and the Cost of Living

## 3.1 Introduction

[1] [2] Over the past decades, a large amount of research has been done to understand the role of creative destruction on long-term growth (Grossman and Helpman (1991), Aghion and Howitt (1992), Klette and Kortum (2004), among others) and business cycle fluctuations (Caballero and Hammour (1996), Foster et al. (2001) Ghironi and Melits (2005), among others). However, the work to empirically quantify the impact of product creation and destruction on the cost of living and consumer welfare is very limited. This gap is partly due to the lack of product-level price and quantity data at a national scale, as well as a theoretical framework which incorporates not only product turnover, but also time-varying demand (taste) shocks at the product level.

Measurement plays an important role in evaluating impact of innovation on welfare. The impact of innovation on welfare depends critically on taking into

[1]The results in this chapter are based on researchers own analyses calculated (or derived) based in part on data from The Nielsen Company (US), LLC marketing databases provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the Nielsen data are those of the researchers and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

[2]This chapter draws heavily from collaborative work from Ehrlich et al. (2020). It also builds on closely related work from Ehrlich et al. (2019a) and Ehrlich et al. (2019b).

account the impact of innovation on the cost of living. One method is hedonics. In practice, hedonics is only applied to a relatively small fraction of goods by BLS (about 5 percent). Also, hedonics is an imperfect method for this purpose. As Pakes (2003) argues hedonics may provide better bounds than traditional price indices but not exact price index. Recent work of Redding and Weinstein (2020) (RW) develops a measurement method that is demand theory based and takes into account not only product turnover (which we have known how to do since Feenstra (1994)) but also changing relative valuation of quality/demand for continuing goods. Redding and Weinstein find that consumer valuation bias in traditional price indices is typically negative and they argue this is intuitive.

In this chapter, I utilize the Unified Price Index (UPI) framework proposed by Redding and Weinstein (2020) and the Nielsen Retail Scanner Dataset (RMS) [3] to measure the evolution of the cost of living with the presence of product creation and destruction, firm entry and exit, and relative taste shocks to continuing goods in the consumer goods sector in the United States. I show that in the UPI framework, the adjustment to the traditional Sato-Vartia index is correlated with the initial period size of a product, different from RW's argument that the adjustment is always negative. I then construct the UPI under a nested CES demand structure to estimate the cost of living in the consumer goods sector and study the contribution to living costs by firms of difference sizes.

A key contribution of Redding and Weinstein (2020) is that they argue with

[3] My results are based on Nielsen Scanner datasets from the Kilts Center at the University of Chicago Booth School of Business. More information can be found from https://www.chicagobooth.edu/research/kilts/datasets/nielsen.

the existence of relative taste shocks, there is a downward adjustment - the consumer valuation adjustment - to the Sato-Vartia index, since increases in tastes are weighted more than reductions in tastes. This is equivalent to the effect of an increase in the dispersion of market shares. I argue, instead, that this adjustment is not always negative. In other words, the Sato-Vartia index is not always upward biased. I first show analytically that in a two-good economy, the direction of the consumer valuation adjustment is determined by the correlation between initial period market shares and taste shocks. Only when taste shocks are positively correlated with initial period market shares, the consumer valuation adjustment is negative. I then test the implications of this result using the Nielsen RMS data and show that this conclusion holds in the multi-good economy. This result implies that positive relative taste shocks to large share goods will reduce the cost of living but the same shock to small share goods will increase the cost of living, other things equal.

The dependence of the consumer valuation adjustment on the initial market share motivates me to explore the contributions to the cost of living by firms of different sizes. I construct the UPI under a nested CES demand system where I assume goods within each firm are equally substitutable and there are product-level demand shocks. A consumer can also substitute goods between firms and the elasticity of substitution is different within and between firms.

The measured inflation by the nested UPI from the Nielsen RMS shows negative adjustments to traditional price indices, due to product turnover and relative taste shocks over time. The measured average annual inflation by the nested UPI is -1.0% from 2006 to 2015, compared with 0.03% given by the Laspeyres index. The measured UPI inflation rates are 2.0% and -3.1% for food and non-food respectively, compared to 3.0% and 0.0% from the Laspeyres index.

Large firms drive down the cost of living in the aggregate economy by driving down the consumer valuation adjustment. Without top 5 firms from each product group, the cost of living would have increased by 14% from 2006 to 2015, or 1.4% on an annual basis. Meanwhile, the role of large firms is different across product groups.

In the innovation intensive sector (eg. electronics), large firms engage in more active product creation and destruction that drives down the cost of living. On the other hand, the dispersion of market shares across products in large firms decreases over time, introducing a positive consumer valuation adjustment. In the less innovation intensive sector (eg. snacks), there is limited product creation and destruction. Moreover, the dispersion of product expenditure within large firms in this sector increases over time, hence drives down the cost of living.

This chapter is closely related to the literature which studies the estimation of exact CES price indices. Feenstra (1994) proposes an adjustment to traditional price indices (Feenstra-adjustment) to take into account product turnover effects. Broda and Weinstein (2006) estimate a CES exact index for import price considering the introduction of new varieties. Broda and Weinstein (2010) first attempted to use item-level transaction datasets to estimate a cost-of-living index. Redding and Weinstein (2020) formally propose the UPI framework and introduce the consumer valuation bias to the traditional Sato-Vartia index. Argente et al. (2019b) use the approach from Redding and Weinstein (2020) to study the cost of living in

Mexico and the US with Nielsen Consumer Panel data. This chapter contributes to the literature by showing the relationship between the initial product size and the direction of the consumer valuation bias in Redding and Weinstein (2020). Using the UPI framework and the Nielsen Retail Scanner dataset, I estimate that the average inflation rate in the consumer goods sector in the United States is should be -1.0% from 2006 to 2015.

More broadly, this chapter is related to the literature which aim to incorporate quality updating to price indices using hedonic approaches. The papers include Feenstra (1995), Diewert (2003), Pakes (2003), Benkard and Bajari (2005), Erickson and Pakes (2011), Diewert et al. (2017), among others. Estimated prices indices from the Nielsen RMS in this chapter compliment to the estimates from hedonic approaches.

This chapter is also closely related to the literature that utilizes transaction-level price and quantity data to measure economics activities and firm behaviors. Ehrlich et al. (2019a) compare the BLS CPI to alternative price indices such as the Feenstra (1994) price index and the UPI constructed from Nielsen Retail Scanner Data. Ehrlich et al. (2019b) explore in more detail the UPI and hedonic approaches for estimating price and quantity indices in large-scale item-level data. Ehrlich et al. (2019b) also discuss opportunities and challenges statistical agencies are facing in this new century. On the firm dynamics front, Argente et al. (2019a) match the Nielsen RMS data to USPTO to study firms innovation behavior and the difference between large and small firms. This chapter estimates the cost of living in the consumer good sector and studies the contribution of large and small firms by constructing the UPI under a nested CES structure.

## 3.2 Data

The data source is the Nielsen Retail Scanner data (also referred to as RMS) from the Kilts Center at the University of Chicago Booth School of Business for the 2006 to 2015 period. The original data consists of over 2.6 million products identified by the finest level of aggregation - 12-digit universal product code (UPC) that uniquely identify specific goods.[4] The RMS data are collected from over $40,000$ individual stores from approximately 90 retail chains in over 370 MSAs in the US. Total sales in Nielsen RMS are worth around $2 trillion and represent 53% of all sales in grocery stores, 55% in drug stores, 32% in mass merchandisers and 2% in convenience stores.

Nielsen organizes barcode-level goods into 10 department, 115 product groups[5] and over 1000 product modules. A typical department is, for example, dry grocery, which consists of 41 product groups such as baby food, coffee and carbonated beverage. Within the carbonated beverage product group, there are product modules such as soft drinks and fountain beverage. In the analysis later, I construct product group level price indices and then aggregate them to form an aggregate price index for the the consumer goods sector. The choice of product group for lower level aggregation is based on the tradeoff between the substitutability between goods and the

[4] In the Nielsen data, there are both UPC code and UPC version code. The unique product identifier used in my analysis is the combination of UPC and UPC version code.

[5] I keep 106 product groups for which elasticities of substitution can be estimated for later analysis.

sample size for firm-level analysis within a group. The product groups are classified into food and non-food sectors based on the BLS correspondence (Table 3.1 and 3.2 list the product groups in food and non-food respectively).

The RMS consists of more than 100 billion unique observations at the week $\times$ store $\times$ UPC level. I first aggregate the weekly data into monthly according to the National Retail Federation (NRF) calendar[6] and then aggregate the monthly data to quarterly.[7] The NRF calendar is a guide for retailers that ensures sales comparability between years by dividing a year into months based on a 4 weeks - 5 weeks - 4 weeks format. The layout of the calendar lines up holidays and ensures the same number of Saturdays and Sundays in comparable months. The NRF calendar ensures the comparability of the aggregated sales over time.

To identify manufacturers of UPCs, I use the matching between UPC code and firm identifier provided by from GS1 US which is the single official source of UPCs. After matching RMS to GS1, I obtain a sample of over 47,000 firms.

Table 3.1 and Table 3.2 provide summary statistics of the 2015 Q3 snapshot for each product group. The median number of UPC in a product group is 2767, and the median number of firms in a product group is 340. The average number of UPC per firm ranges from 14 at the 5th percentile to 430 at the 95th percentile. Average firm sales in a product group range from \$1.4 million at the 5th percentile to \$68 million at the 95th percentile. In terms of average UPC sales, they are \$16,000 at the 5th percentile, and \$290,000 at the 95th percentile.

[6] NRF calendars are collected from the NRF website: https://nrf.com/resources/4-5-4-calendar

[7] Before aggregating to quarterly, I drop outliers, defined as the observations with price above 3 times median or below 1/3 of median for each UPC in a given month. I also drop observations whose quantity is about 24 times of the median quantity in a given month.

| | $N_{upc}$ | $N_{upc}$ per Firm | $N_{firm}$ | Avg. Firm Sales | Avg. UPC Sales |
|---|---|---|---|---|---|
| Baby food | 1,484 | 38 | 185 | 51,800 | 319 |
| Candy | 17,533 | 1,247 | 164 | 39,900 | 76 |
| Fruit - canned | 1,185 | 246 | 22 | 2,268 | 59 |
| Gum | 1,141 | 114 | 202 | 52,200 | 177 |
| Jams, jellies, spreads | 3,622 | 609 | 30 | 8,012 | 64 |
| Juice, drinks - canned, bottled | 7,000 | 789 | 58 | 25,100 | 192 |
| Prepared food-ready-to-serve | 4,748 | 807 | 41 | 7,304 | 95 |
| Prepared food-dry mixes | 3,171 | 485 | 38 | 12,000 | 146 |
| Seafood - canned | 1,624 | 239 | 33 | 7,454 | 88 |
| Soup | 2,957 | 395 | 90 | 36,300 | 145 |
| Baking mixes | 2,040 | 281 | 48 | 8,262 | 75 |
| Breakfast food | 1,972 | 198 | 77 | 31,200 | 202 |
| Cereal | 2,574 | 295 | 97 | 55,600 | 303 |
| Coffee | 4,816 | 442 | 68 | 25,100 | 177 |
| Desserts, gelatins, syrup | 1,606 | 285 | 30 | 5,304 | 99 |
| Flour | 861 | 204 | 18 | 1,448 | 55 |
| Fruit - dried | 2,232 | 291 | 27 | 3,857 | 66 |
| Nuts | 4,125 | 400 | 45 | 6,414 | 67 |
| Packaged milk and modifiers | 1,200 | 214 | 35 | 21,700 | 234 |
| Pasta | 3,356 | 370 | 36 | 3,347 | 44 |
| Pickles, olives, and relish | 3,644 | 477 | 33 | 2,281 | 37 |
| Salad dressings, mayo, toppings | 2,339 | 384 | 47 | 11,700 | 120 |
| Shortening, oil | 1,776 | 405 | 14 | 2,591 | 98 |
| Spices, seasoning, extracts | 11,535 | 1,123 | 113 | 5,631 | 20 |
| Sugar, sweeteners | 705 | 204 | 13 | 4,027 | 137 |
| Table syrups, molasses | 731 | 210 | 8 | 689 | 58 |
| Tea | 5,023 | 545 | 54 | 8,012 | 83 |
| Vegetables and grains - dried | 2,077 | 322 | 29 | 2,551 | 42 |
| Bread and baked goods | 14,494 | 1,284 | 90 | 9,334 | 93 |
| Carbonated beverages | 6,556 | 477 | 232 | 141,000 | 290 |
| Cookies | 6,172 | 871 | 50 | 17,300 | 82 |
| Crackers | 2,142 | 365 | 33 | 22,300 | 197 |
| Snacks | 16,431 | 1,623 | 144 | 73,200 | 119 |
| Soft drinks-non-carbonated | 4,343 | 896 | 41 | 10,400 | 158 |
| Baked goods-frozen | 1,299 | 278 | 15 | 2,482 | 95 |
| Breakfast foods-frozen | 1,030 | 178 | 22 | 12,100 | 231 |
| Ice cream, novelties | 5,883 | 319 | 85 | 14,200 | 126 |
| Juices, drinks-frozen | 196 | 34 | 14 | 1,764 | 88 |
| Pizza/snacks/hors doeurves-frzn | 2,905 | 491 | 29 | 15,600 | 173 |
| Prepared foods-frozen | 7,216 | 1,035 | 42 | 17,100 | 155 |
| Unprep meat/poultry/seafood-frzn | 2,441 | 484 | 24 | 2,004 | 82 |
| Vegetables-frozen | 2,125 | 214 | 71 | 12,500 | 129 |
| Butter and margarine | 590 | 167 | 13 | 9,293 | 368 |
| Cheese | 7,213 | 571 | 103 | 25,000 | 120 |
| Cot cheese, sour cream, toppings | 1,395 | 214 | 20 | 2,206 | 117 |
| Dough products | 391 | 77 | 66 | 42,200 | 329 |
| Eggs | 613 | 165 | 9 | 4,340 | 265 |
| Milk | 3,829 | 287 | 46 | 11,400 | 162 |
| Pudding, desserts-dairy | 275 | 61 | 17 | 4,399 | 134 |
| Snacks, spreads, dips-dairy | 2,640 | 575 | 24 | 1,976 | 72 |
| Yogurt | 2,976 | 180 | 94 | 38,800 | 241 |
| Dressings/salads/prep foods-deli | 8,134 | 1,156 | 79 | 7,836 | 120 |
| Packaged meats-deli | 8,206 | 683 | 129 | 15,400 | 161 |
| Fresh meat | 1,011 | 218 | 24 | 4,539 | 257 |
| Fresh produce | 9,302 | 1,277 | 166 | 37,300 | 145 |

Table 3.1: Summary statistics of food product groups. Snapshot at 2015 Q3. Quarterly sales in thousands

| | $N_{upc}$ | $N_{upc}$ per Firm | $N_{firm}$ | Avg. Firm Sales | Avg. UPC Sales |
|---|---|---|---|---|---|
| Pet food | 5,150 | 282 | 182 | 62,600 | 189 |
| Ice | 383 | 201 | 5 | 1,776 | 147 |
| Detergents | 2,634 | 115 | 459 | 220,000 | 318 |
| Disposable diapers | 1,397 | 19 | 494 | 162,000 | 286 |
| Fresheners and deodorizers | 5,375 | 302 | 168 | 14,000 | 41 |
| Household cleaners | 2,631 | 419 | 46 | 12,300 | 122 |
| Household supplies | 6,146 | 705 | 74 | 4,611 | 50 |
| Laundry supplies | 2,827 | 369 | 76 | 19,300 | 124 |
| Paper products | 10,752 | 482 | 377 | 52,400 | 121 |
| Personal soap and bath additives | 6,191 | 545 | 125 | 19,300 | 70 |
| Pet care | 9,317 | 614 | 143 | 3,139 | 32 |
| Tobacco & accessories | 6,774 | 342 | 167 | 99,800 | 245 |
| Wrapping materials and bags | 1,545 | 218 | 82 | 28,500 | 171 |
| Beer | 12,210 | 1,296 | 65 | 34,900 | 130 |
| Liquor | 13,550 | 866 | 291 | 18,700 | 61 |
| Wine | 22,734 | 2,790 | 79 | 10,800 | 55 |
| Automotive | 1,696 | 190 | 32 | 1,573 | 36 |
| Batteries and flashlights | 2,514 | 289 | 212 | 35,400 | 89 |
| Books and magazines | 726 | 39 | 42 | 5,010 | 106 |
| Canning, freezing supplies | 206 | 46 | 26 | 4,979 | 130 |
| Charcoal, logs, accessories | 881 | 241 | 19 | 3,936 | 75 |
| Electronics, records, tapes | 23,746 | 452 | 977 | 14,100 | 14 |
| Floral, gardening | 972 | 120 | 58 | 870 | 10 |
| Glassware, tableware | 18,237 | 744 | 200 | 1,548 | 8 |
| Hardware, tools | 4,523 | 503 | 153 | 11,400 | 39 |
| Housewares, appliances | 6,960 | 468 | 126 | 10,800 | 80 |
| Insecticds/pesticds/rodenticds | 1,662 | 222 | 52 | 5,935 | 50 |
| Kitchen gadgets | 19,261 | 1,276 | 259 | 3,698 | 12 |
| Light bulbs, electric goods | 5,294 | 373 | 340 | 15,700 | 30 |
| Photographic supplies | 344 | 50 | 24 | 2,206 | 86 |
| Sewing notions | 403 | 100 | 21 | 676 | 28 |
| Shoe care | 569 | 46 | 81 | 2,413 | 16 |
| Stationery, school supplies | 22,846 | 1,027 | 242 | 9,535 | 24 |
| Baby needs | 3,847 | 338 | 80 | 5,821 | 62 |
| Cosmetics | 25,850 | 396 | 1,002 | 32,700 | 25 |
| Cough and cold remedies | 2,002 | 199 | 46 | 15,600 | 259 |
| Deodorant | 1,666 | 80 | 202 | 47,600 | 173 |
| Diet aids | 282 | 56 | 28 | 3,942 | 116 |
| Ethnic haba | 407 | 58 | 24 | 1,103 | 38 |
| Feminine hygiene | 392 | 89 | 15 | 3,353 | 146 |
| First aid | 2,707 | 485 | 44 | 5,292 | 71 |
| Fragrances - women | 4,725 | 346 | 142 | 1,900 | 12 |
| Grooming aids | 10,061 | 515 | 530 | 12,600 | 19 |
| Hair care | 14,412 | 474 | 430 | 24,200 | 57 |
| Medications/remedies/health aids | 11,102 | 1,079 | 82 | 11,100 | 112 |
| Men's toiletries | 1,523 | 177 | 46 | 828 | 20 |
| Oral hygiene | 2,985 | 229 | 167 | 49,200 | 197 |
| Sanitary protection | 1,015 | 27 | 174 | 53,400 | 255 |
| Shaving needs | 1,365 | 126 | 173 | 68,200 | 234 |
| Skin care preparations | 7,366 | 665 | 101 | 12,200 | 66 |
| Vitamins | 10,941 | 952 | 116 | 11,600 | 74 |

Table 3.2: Summary statistics of non-food product groups. Snapshot at 2015 Q3. Quarterly sales in thousands.

One feature of barcode (UPC) is that goods of different size and packaging have different barcodes. To ensure the comparability between prices, I follow Hottman et al. (2016) and normalize UPC prices to the same unit (oz), utilizing the size and packaging information provided by Nielsen. To deal with outlier issues, I winsorize the price change at the top and bottom 1% of each product group.

## 3.3 Measuring the Exact Cost of Living under Product Creative Destruction and Relative Taste Shocks: the Unified Price Index Framework

Redding and Weinstein (2020) propose an Unified Price Index (UPI) framework under CES demand to measure the exact cost of living when there are product turnover and relative taste shocks. In this section, I first describe the flat UPI proposed in RW. Then, in a two-good economy, I show analytically that the direction of the consumer valuation bias, the critical component in RW, depends on the correlation between relative taste shocks and the initial period size of a product, different from RW's argument that relative taste shocks will always lead to a downward adjustment to the Sato-Vartia index. I then consider a nested CES structure where there are item-level demand shocks, goods entry and exit, and firm entry and exit to study the contribution to the living cost by firms of difference sizes.

### 3.3.1 The Flat UPI

In a given sector (product group), the consumer preference is defined as

$$C_{gt} = \left( \sum_{u \in \Omega_{gt}^U} (\varphi_{ugt} C_{ugt})^{\frac{\sigma_g^U - 1}{\sigma_g^U}} \right)^{\frac{\sigma_g^U}{\sigma_g^U - 1}}. \tag{3.1}$$

The unit expenditure function for this utility function is given by

$$P_{gt} = \left[ \sum_{u \in \Omega_{gt}^U} \left( \frac{P_{ugt}}{\varphi_{ugt}} \right)^{1-\sigma_g^U} \right]^{\frac{1}{1-\sigma_g^U}}, \tag{3.2}$$

where $u$ indicates goods (UPC), $g$ indicates product group. $\Omega_{gt}^U$ is the set of goods in product group $g$ that have positive sales at time $t$. $\varphi_{ugt}$ is the appeal of good $u$, and $\sigma_g^U$ is the elasticity of substitution within product group $g$.

Under the normalization assumption that the geometric average of product appeal $\varphi_{ugt}$ is constant over time, the change in the unit expenditure function can be written as

$$\Phi_{t-1,t}^{g,flat} = \frac{P_{gt}}{P_{gt-1}} = \left( \frac{\lambda_{gt,t-1}^u}{\lambda_{gt-1,t}^u} \right)^{\frac{1}{\sigma_g^U - 1}} \cdot \frac{\tilde{P}_{gt}^*}{\tilde{P}_{gt-1}^*} \left( \frac{\tilde{S}_{gt}^*}{\tilde{S}_{gt-1}^*} \right)^{\frac{1}{\sigma_g^U - 1}}, \tag{3.3}$$

where

$$\lambda_{gt,t-1}^u \equiv \frac{\sum_{u \in \Omega_{gt,t-1}^U} (P_{ugt}/\varphi_{ugt})^{1-\sigma_g^U}}{\sum_{u \in \Omega_{gt}^U} (P_{ugt}/\varphi_{ugt})^{1-\sigma_g^U}}, \quad \lambda_{gt-1,t}^u \equiv \frac{\sum_{u \in \Omega_{gt,t-1}^U} (P_{ugt-1}/\varphi_{ugt-1})^{1-\sigma_g^U}}{\sum_{u \in \Omega_{gt-1}^U} (P_{ugt-1}/\varphi_{ugt-1})^{1-\sigma_g^U}}, \tag{3.4}$$

and

$$\frac{\tilde{S}^*_{gt}}{\tilde{S}^*_{gt-1}} = \frac{\prod_{u\in\Omega^U_{gt,t-1}} \left(S^*_{ugt}\right)^{1/N_{t,t-1}}}{\prod_{u\in\Omega^U_{gt,t-1}} \left(S^*_{ugt-1}\right)^{1/N_{t,t-1}}}, \tag{3.5}$$

$$\frac{\tilde{P}^*_{gt}}{\tilde{S}^*_{gt-1}} = \frac{\prod_{u\in\Omega^U_{gt,t-1}} \left(P^*_{ugt}\right)^{1/N_{t,t-1}}}{\prod_{u\in\Omega^U_{gt,t-1}} \left(P^*_{ugt-1}\right)^{1/N_{t,t-1}}}. \tag{3.6}$$

$\Phi^{g,flat}_{t-1,t}$ is the exact price index under the CES preference, and is called the Unified Price Index (UPI) by Redding and Weinstein (2020). Equation 3.5 is the consumer valuation (CV) bias term, 3.6 is the common goods price relative, or the Jevons index, and the term $\left(\frac{\lambda^u_{gt,t-1}}{\lambda^u_{gt-1,t}}\right)^{\frac{1}{\sigma^U_g-1}}$ in 3.3 is the Feenstra-adjustment term for UPC entry and exit.

### 3.3.2 Initial Period Market Share and the Direction of Consumer Valuation Bias

A key contribution of Redding and Weinstein (2020) is that they argue that the Sato-Vartia index is upward biased when there are relative demand shocks cross goods. The bias is measured by the consumer valuation bias term (equation 3.5). While RW argue that increases in tastes are weighted more than reductions in tastes hence relative taste shocks increase dispersion in market shares and result in a upward bias in the Sato-Vartia index, I show in this section, in a two-good economy, that the Sato-Vartia index is not always upward biased. The direction of the bias is determined by the correlation between the initial period market share and the taste shock. When the relative appeal of the high market share good ("winning" good) increases, the consumer valuation bias adjustment will be negative (hence the Sato-Vartia index is upward biased). However, if the relative appeal of the low market share good increases, the consumer valuation bias adjustment will be positive (hence the Sato-Vartia index is downward biased).

#### 3.3.2.1 A Two-Good Economy

Suppose there are only two goods in the economy and the consumer preference takes the form

$$C_t = \left( \sum_{u=1,2} (\varphi_{ut} C_{ut})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3.7}$$

and output is

$$Y_t = p_{1t} C_{1t} + p_{2t} C_{2t}.$$

The first order equation of the consumer optimization problem gives:

$$s_{ut} = \frac{p_{ut} C_{ut}}{Y_t} = \frac{(p_{ut}/\varphi_{ut})^{1-\sigma}}{\sum_{u=1,2} (p_{ut}/\varphi_{ut})^{1-\sigma}}. \tag{3.8}$$

The exact cost function is hence

$$\begin{aligned} UPI_t &= \frac{Y_t}{\left( \sum_{u=1,2} (\varphi_{ut} C_{ut})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}} \\ &= \left( \sum_{u=1,2} \left(\frac{p_{ut}}{\varphi_{ut}}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \end{aligned} \tag{3.9}$$

Now consider the case where prices remain constant, i.e. $p_{ut} = p_{ut-1}$ and there are taste shocks to $\varphi_{ut}$, i.e. $\varphi_{ut} \neq \varphi_{ut-1}$. Since prices of goods do not change, a conventional price index, whether it is Laspeyres, Paasche, or Sato-Vartia, doesn't change between $t-1$ to $t$.

To explore how the UPI will change, define

$$\begin{aligned} M_t &= (p_{1t}p_{2t})^{1-\sigma} = (p_{1t-1}p_{2t-1})^{1-\sigma}, \\ y_{ut} &= \big(\frac{p_{it}}{\varphi_{it}}\big)^{1-\sigma} \end{aligned}$$

Normalization of taste shocks gives

$$\varphi_{1t}\varphi_{2t} = 1, \tag{3.10}$$

for all $t$. From equation 3.8, we have

$$\frac{y_{1t}}{y_{2t}} = \frac{s_{1t}}{s_{2t}}. \tag{3.11}$$

Now equation 3.9 can be written as

$$\begin{aligned} UPI_t &= (y_{1t} + y_{2t})^{\frac{1}{\sigma-1}} \\ &= M^{\frac{1}{2(1-\sigma)}}\Big(\big(\frac{s_{1t}}{s_{2t}}\big)^{0.5} + \big(\frac{s_{2t}}{s_{1t}}\big)^{0.5}\Big)^{\frac{1}{1-\sigma}} \\ &= (p_{1t}p_{1t})^{0.5}\Big(\big(\frac{s_{1t}}{s_{2t}}\big)^{0.5} + \big(\frac{s_{2t}}{s_{1t}}\big)^{0.5}\Big)^{\frac{1}{1-\sigma}}, \end{aligned} \tag{3.12}$$

and in log form, we have

$$\ln UPI_t = 0.5 \ln p_{1t} p_{1t} + \frac{1}{1-\sigma} \ln \Big( \big(\frac{s_{1t}}{s_{2t}}\big)^{0.5} + \big(\frac{s_{2t}}{s_{1t}}\big)^{0.5} \Big). \tag{3.13}$$

The second term in this equation is the consumer valuation bias.

Without loss of generality, assume $s_{1t-1}/s_{2t-1} > 1$. When $\sigma > 1$, the function $f(x_t) = \Big( x_t^{0.5} + \big(\frac{1}{x_t}\big)^{0.5} \Big)^{\frac{1}{1-\sigma}}$, $x = s_{1t}/s_{2t}$, is deceasing with respect to $x$. Therefore $f(x_t) < f(x_{t-1})$ if and only if $s_{1t}/s_{2t} > s_{1t-1}/s_{2t-1}$, since $s_{1t} + s_{2t} = 1$, this is equivalent to say $f(x_t) < f(x_{t-1})$ if and only if $s_{1t} > s_{1t-1}$. So we have the following proposition.

**Proposition 1:** In a two-good economy with the elasticity of substitution $\sigma > 1$, and only relative taste shocks, assume $s_{1t-1} > s_{2t-1}$, $UPI_t < UPI_{t-1}$ if and only if $s_{1t} > s_{1t-1}$, where $s_{ut}$ is expenditure share of good $u$ at time $t$.

The intuition is that if it is easy for the representative consumer to substitute goods, i.e. $\sigma > 1$, the living cost is lower if the dispersion of the quality adjusted price is higher, making it more effective for a substitution of cheap goods for expensive goods.

This intuition can be generalized to an economy with more than two goods. Given the consumer valuation bias is a concave function of the expenditure share, what is behind the two-good result is a change in the dispersion of the expenditure shares. More specifically, a larger dispersion is associated with a lower cost of living, because it implies a stronger substitution of relatively cheap goods (those with large expenditure shares) for relatively expensive goods (those with small expenditure shares)–this substitution is not captured by traditional indexes but by the universal price index. This force clearly does not depend on the number of goods. For example, if the good with the largest expenditure further gains expenditure shares and the rest of the goods, whose number can exceed two, do not experience a significant change in their relative market shares, we may expect the same consequence of higher dispersion in expenditure shares and a decline in the consumer valuation bias. The key to emphasize is that it is not the change in expenditure share that matters for the consumer valuation bias, but what the initial expenditure share is for those that indeed experience a change in their expenditure share.

Proposition 1 is consistent with Redding and Weinstein (2020)'s argument that an increase in the dispersion in expenditure shares will lead to a decrease in the consumer valuation bias. However, the difference between Proposition 1 and Redding and Weinstein (2020) is that RW regard relative taste shocks across goods to be always dispersion enhancing, hence the consumer valuation bias term is always negative under relative taste shocks. Proposition 1, instead, says that consumer valuation bias is negative if and only if the relative appeal of the large share good continue to increase (or equivalently small goods continue to shrink).

This proposition also suggests that innovation from small and large firms may have different implications for the aggregate cost of living. If small firms have successful innovations but do not lower their prices, they will gain market share and reduce the dispersion of the quality-adjusted price, which is not good for the living cost to decline. In contrast, successful innovation from large firms can reduce the consumer valuation bias and lower the cost of living.

These results motivate me to study the contribution of firms of different sizes to the living cost, which arguably can help reveal firms' innovation and pricing behaviors.

#### 3.3.2.2 Empirical Tests for Proposition 1

To test the predictions of Proposition 1, I first back out relative taste shocks from the data (the derivation will be explained in section 3.3.5) and run following regressions for each product group:

$$\ln \frac{\varphi_{ut}}{\varphi_{ut-1}} = c_1 + a_g \ln s_{ut-1} + I_t + I_u + \epsilon_{ut} \tag{3.14}$$

$$\ln \frac{\varphi_{ut}}{\varphi_{ut-1}} = c_2 + b_g \ln \frac{s_{ut}}{s_{ut-1}} + I_t + I_u + \delta_{ut}, \tag{3.15}$$

where $s_{ut}$ is the expenditure share of good $u$ at time $t$. $I_t$ and $I_u$ are time fixed effects and product fixed effects respectively.

The second step is to collect the estimated coefficients $a_g$ and $b_g$ for all product groups and regress the consumer valuation bias (CV) on $a_g$ and $b_g$ separately:

$$CV_g = c_3 + \beta \times a^g + \kappa_g \tag{3.16}$$

$$CV_g = c_4 + \gamma \times b^g + \nu_g. \tag{3.17}$$

If Proposition 1 holds in the data, we should expect $\beta$ to be negative, i.e. when relative taste shocks are correlated with initial period market shares, the consumer valuation adjustment is negative. If the force described by Redding and Weinstein

(2020) holds in the data, i.e. changes in expenditure shares are positively correlated with relative taste shocks, we should expect $\gamma$ to be negative.

### 3.3.3 The UPI under Nested CES Demand

Now I derive the nested UPI under the CES demand similar to Hottman et al. (2016) (HRW). In the baseline case, I assume there are only item-level taste shocks. Then in the full case, I add firm-level taste shocks.

The demand is a two-tier CES structure which has the following form:

$$C_{gt} = \left( \sum_{f \in \Omega_{gt}^F} C_{fgt}^{\frac{\sigma_g^F - 1}{\sigma_g^F}} \right)^{\frac{\sigma_g^F}{\sigma_g^F - 1}}, \tag{3.18}$$

and

$$C_{fgt} = \left( \sum_{u \in \Omega_{fgt}^U} \left( \varphi_{ufgt} C_{ufgt} \right)^{\frac{\sigma_g^U - 1}{\sigma_g^U}} \right)^{\frac{\sigma_g^U}{\sigma_g^U - 1}}. \tag{3.19}$$

The real consumption in any product group, $g$, is a function of the consumption of each firm $f$'s output, $C_{fgt}$, and adjusted for the substitutability of the output of each firm. The elasticity of substitution between firms, denoted by $\sigma_g^F$, is greater than 1. The subutility derived from the consumption of a firm $f$'s output within product group $g$ is a function of the quantity consumed of each UPI (barcode) $u$, $C_{ufgt}$, weighted by the consumer appeal of that UPC's output, $\varphi_{ufgt} > 0$, and adjusted for the substitutability between barcodes supplied by the firm. The elasticity of substitution within firms, denoted by $\sigma_g^U$, is greater than 1. The set of firms that are active at time $t$ is denoted by $\Omega_{gt}^F$, and the set of active products of firm $f$ at time

$t$ is denoted by $\Omega^U_{fgt}$. For future reference, it is useful to define the set of products within firm $f$ that exist throughout the time period $t-1, t$, i.e the common goods, as $\Omega^U_{fgt,t-1}$. This nested specification allows for the introduction of new UPCs to firms and new firms to each product group.

The unit expenditure function for utility function 3.19 is given by

$$P_{fgt} = \left[ \sum_{u \in \Omega^U_{fgt}} \left( \frac{P_{ufgt}}{\varphi_{ufgt}} \right)^{1-\sigma^U_g} \right]^{\frac{1}{1-\sigma^U_g}}, \tag{3.20}$$

and the unit expenditure function for 3.18 is

$$P_{gt} = \left[ \sum_{f \in \Omega^F_{gt}} \left( P_{fgt} \right)^{1-\sigma^F_g} \right]^{\frac{1}{1-\sigma^F_g}}. \tag{3.21}$$

Applying Shephard's Lemma, we obtain the expenditure share for each good in firm $f$ is:

$$S_{ufgt} = \frac{P_{ufgt} C_{ufgt}}{\sum_{u \in \Omega^U_{ft}} P_{ufgt} C_{ufgt}} = \frac{(P_{ufgt}/\varphi_{ufgt})^{1-\sigma^U_g}}{P_{fgt}^{1-\sigma^U_g}}, \tag{3.22}$$

and the expenditure share for each firm in product group $g$ is

$$S_{fgt} = \frac{P_{fgt} C_{fgt}}{\sum_{f \in \Omega^F_t} P_{fgt} C_{fgt}} = \frac{(P_{fgt})^{1-\sigma^F_g}}{P_{gt}^{1-\sigma^F_g}}. \tag{3.23}$$

#### 3.3.3.1 Entry and Exit

Feenstra (1994) shows that under CES preference, there is a tractable way of correcting the entry and exit of new product varieties in price indices. I implement this variety correction at both product and firm level.

Within a firm $f$, we can partition the set of UPCs active at time $t$ into the set of goods that are common in both $t-1$ and $t$, $\Omega^U_{fgt,t-1}$, and the set of goods that enter between period $t-1$ to $t$. Similarly, we can partition the set of UPCs active at time $t-1$ into the set of goods that are common in both $t-1$ and $t$, $\Omega^U_{fgt,t-1}$, and the set of good that exit between period $t-1$ to $t$.

Within a product group $g$, there can be firm entry and exit. Denote the firms that are common between $t-1$ and $t$ by $\Omega^F_{gt,t-1}$. We can partition $\Omega^F_{gt}$ into common firms,$\Omega^F_{gt,t-1}$, and firms that enter during $t-1$ to $t$. Similarly, the set of firms that are active at $t-1$ ($\Omega^F_{gt-1}$) can be partitioned into $\Omega^F_{gt,t-1}$ and firms that exit during $t-1$ to $t$.

Using this notation, the change in the exact price index within a firm $f$ in period $t-1$ to $t$, $\Phi_{t-1,t}$ can be written as

$$\Phi^{fg}_{t-1,t} \equiv \frac{P_{fgt}}{P_{fgt-1}} = \left(\frac{\lambda^u_{fgt,t-1}}{\lambda^u_{fgt-1,t}}\right)^{\frac{1}{\sigma^U_g-1}} \cdot \frac{P^*_{fgt}}{P^*_{fgt-1}}, \tag{3.24}$$

where the $\left(\lambda^u_{fgt,t-1}/\lambda^u_{fgt-1,t}\right)^{\frac{1}{\sigma^U_g-1}}$ is the Feenstra variety-adjustment term and is defined as

$$\lambda^u_{fgt,t-1} \equiv \frac{\sum_{u\in\Omega^U_{fgt,t-1}}(P_{ufgt}/\varphi_{ufgt})^{1-\sigma^U_g}}{\sum_{u\in\Omega^U_{fgt}}(P_{ufgt}/\varphi_{ufgt})^{1-\sigma^U_g}}, \quad \lambda^u_{fgt-1,t} \equiv \frac{\sum_{u\in\Omega^U_{fgt,t-1}}(P_{ufgt-1}/\varphi_{ufgt-1})^{1-\sigma^U_g}}{\sum_{u\in\Omega^U_{fgt-1}}(P_{ufgt-1}/\varphi_{ufgt-1})^{1-\sigma^U_g}}. \tag{3.25}$$

Common goods prices $P^*_{fgt}$ and $P^*_{fgt-1}$ are defined as

$$P^*_{fgt} \equiv \left[ \sum_{u \in \Omega^U_{fgt,t-1}} \left( \frac{P_{ufgt}}{\varphi_{ufgt}} \right)^{1-\sigma^U_g} \right]^{\frac{1}{1-\sigma^U_g}}, \quad P^*_{fgt-1} \equiv \left[ \sum_{u \in \Omega^U_{fgt,t-1}} \left( \frac{P_{ufgt-1}}{\varphi_{ufgt-1}} \right)^{1-\sigma^U_g} \right]^{\frac{1}{1-\sigma^U_g}}. \tag{3.26}$$

If entering goods are more numerous or have lower quality-adjusted prices than exiting goods, the Feenstra adjustment term is less than 1, and the exact price index will fall.

Given common goods prices, we can define the common goods expenditure share of a good $u$ within firm $f$, $S^*_{ufgt}$ and $S^*_{ufgt-1}$ as

$$\begin{aligned} S^*_{ufgt} &\equiv \frac{P_{ufgt}C_{ufgt}}{\sum_{u \in \Omega^U_{fgt,t-1}} P_{ufgt}C_{ufgt}} = \frac{(P_{ufgt}/\varphi_{ufgt})^{1-\sigma^U_g}}{(P^*_{fgt})^{1-\sigma^U_g}} \\ S^*_{ufgt-1} &\equiv \frac{P_{ufgt-1}C_{ufgt-1}}{\sum_{u \in \Omega^U_{fgt,t-1}} P_{ufgt-1}C_{ufgt-1}} = \frac{(P_{ufgt-1}/\varphi_{ufgt-1})^{1-\sigma^U_g}}{(P^*_{fgt-1})^{1-\sigma^U_g}}, \end{aligned} \tag{3.27}$$

which takes the summation of the common goods instead of all active goods in 3.22.

Within a product group $g$, I allow for the entry and exit of firms. I define the change in the exact price index for a product group $g$ as

$$\Phi^g_{t-1,t} \equiv \frac{P_{gt}}{P_{gt-1}} = \left( \frac{\lambda^f_{gt,t-1}}{\lambda^f_{gt-1,t}} \right)^{\frac{1}{\sigma^F_g - 1}} \cdot \frac{P^*_{gt}}{P^*_{gt-1}}. \tag{3.28}$$

The Feenstra-adjustment term for firm entry and exit is defined as

$$\lambda^f_{gt,t-1} \equiv \frac{\sum_{f \in \Omega^F_{gt,t-1}} (P_{fgt})^{1-\sigma^F_g}}{\sum_{f \in \Omega^F_{gt}} (P_{fgt})^{1-\sigma^F_g}}, \quad \lambda^f_{gt-1,t} \equiv \frac{\sum_{f \in \Omega^F_{gt,t-1}} (P_{fgt-1})^{1-\sigma^F_g}}{\sum_{f \in \Omega^F_{gt-1}} (P_{fgt-1})^{1-\sigma^F_g}}, \tag{3.29}$$

and the common goods (firm) prices are defined as

$$P_{gt}^{*}=\Big[\sum_{f\in\Omega_{gt,t-1}^{F}}\big(P_{fgt}\big)^{1-\sigma_g^F}\Big]^{\frac{1}{1-\sigma_g^F}}\quad P_{gt-1}^{*}=\Big[\sum_{f\in\Omega_{gt,t-1}^{F}}\big(P_{fgt-1}\big)^{1-\sigma_g^F}\Big]^{\frac{1}{1-\sigma_g^F}}. \tag{3.30}$$

The common goods (firms) expenditure share are defined as

$$\begin{aligned} S_{fgt}^{*} &= \frac{P_{fgt}C_{fgt}}{\sum_{f\in\Omega_{gt,t-1}^{F}}P_{fgt}C_{fgt}}=\frac{(P_{fgt})^{1-\sigma_g^F}}{(P_{gt}^{*})^{1-\sigma_g^F}},\\ S_{fgt-1}^{*} &= \frac{P_{fgt-1}C_{fgt-1}}{\sum_{f\in\Omega_{gt,t-1}^{F}}P_{fgt-1}C_{fgt-1}}=\frac{(P_{fgt-1})^{1-\sigma_g^F}}{(P_{gt-1}^{*})^{1-\sigma_g^F}}. \end{aligned} \tag{3.31}$$

#### 3.3.3.2 The Exact Price Index for Common Goods/Firms

The exact price index under taste shocks can be written as a function of observable prices and expenditure shares by applying certain normalization over taste ($\varphi$) shocks following Redding and Weinstein (2020).[8] Following the same approach, the change in the cost of living for common goods within a firm $f$ in product group $g$ can be written as

$$\begin{aligned} \Phi_{t-1,t}^{fg*} &\equiv \frac{P_{fgt}^{*}}{P_{fgt-1}^{*}}\\ &= \frac{\big(\prod_{u\in\Omega_{fgt,t-1}^{U}}P_{ufgt}\big)^{1/N_{t,t-1}}}{\big(\prod_{u\in\Omega_{fgt,t-1}^{U}}P_{ufgt-1}\big)^{1/N_{t,t-1}}}\cdot\left(\frac{\prod_{u\in\Omega_{fgt,t-1}^{U}}\big(S_{ufgt}^{*}\big)^{1/N_{t,t-1}}}{\prod_{u\in\Omega_{fgt,t-1}^{U}}\big(S_{ufgt-1}^{*}\big)^{1/N_{t,t-1}}}\right)^{\frac{1}{\sigma_g^U-1}}\\ &= \frac{\tilde{P}_{fgt}^{*}}{\tilde{P}_{fgt-1}^{*}}\left(\frac{\tilde{S}_{fgt}^{*}}{\tilde{S}_{fgt-1}^{*}}\right)^{\frac{1}{\sigma_g^U-1}}, \end{aligned} \tag{3.32}$$

[8] In particular, Redding and Weinstein (2020) assumes that the geometric average of the taste shocks is zero under the law of large numbers. They also demonstrate the robustness of their results under other normalization assumptions.

where $N_{t,t-1}$ is the number of goods in the common goods set $\Omega^{U}_{fgt,t-1}$, and a tilde over a variable denotes a geometric average of a variable across the common goods set.

Within a product group across firms, the exact price index is the Sato-Vartia (Sato (1976) and Vartia (1976)) index:

$$\begin{aligned}\Phi^{g*}_{t-1,t} &\equiv \frac{P^*_{gt}}{P^*_{gt-1}} \\ &= \prod_{\Omega^F_{gt,t-1}} \left(\frac{P_{fgt}}{P_{fgt-1}}\right)^{\omega_{fgt^*}},\end{aligned} \tag{3.33}$$

where the Sato-Vartia weight, $\omega^*_{fgt}$ is given by

$$\omega^*_{fgt} \equiv \frac{\frac{S^*_{fgt}-S^*_{fgt-1}}{\ln S^*_{fgt}-\ln S^*_{fgt-1}}}{\sum_{f\in\Omega^F_{gt,t-1}}\left(\frac{S^*_{fgt}-S^*_{fgt-1}}{\ln S^*_{fgt}-\ln S^*_{fgt-1}}\right)}. \tag{3.34}$$

### 3.3.3.3 The Nested UPI

Assume the utility at the time $t$ is a Cobb-Douglas aggregate of real consumption $C_{gt}$ of all product groups:

$$\ln \mathbf{U_t} = \sum_{g\in\Omega^G} \alpha_{gt} \ln C_{gt}, \quad \sum_{g\in\Omega^G} \alpha_{gt} = 1, \tag{3.35}$$

where $g$ denotes product group, $\alpha_{gt}$ is the expenditure share of group $g$ at time $t$, and $\Omega^G$ is the set of product groups and it is constant over time. Now I can write down the exact price index for the economy: $\Phi_{t-1,t}$, as

$$\Phi_{t-1,t} \equiv \frac{P_t}{P_{t-1}} = \prod_{g\in\Omega^G} (\Phi^g_{t-1,t})^{\alpha_{gt}} \tag{3.36}$$

$$\Phi^g_{t-1,t} = \left(\frac{\lambda^f_{gt,t-1}}{\lambda^f_{gt-1,t}}\right)^{\frac{1}{\sigma^F_g-1}} \cdot \prod_{f\in\Omega^F_{gt,t-1}} \left(\Phi^{fg}_{t-1,t}\right)^{\omega^*_{fgt}} \tag{3.37}$$

$$\Phi^{fg}_{t-1,t} = \left(\frac{\lambda^u_{fgt,t-1}}{\lambda^u_{fgt-1,t}}\right)^{\frac{1}{\sigma^U_g-1}} \cdot \frac{\tilde{P}^*_{fgt}}{\tilde{P}^*_{fgt-1}} \left(\frac{\tilde{S}^*_{fgt}}{\tilde{S}^*_{fgt-1}}\right)^{\frac{1}{\sigma^U_g-1}}. \tag{3.38}$$

Note that 3.38 is the CES Unified Price Index index as defined in Redding and Weinstein (2020); 3.37 is the Feenstra-adjusted Sato-Vartia index at the firm level.

Under such specification, I can further decompose the exact price index at product group level into the contribution of firm entry and exit, goods (UPC) entry and exit, common goods price changes, consumer valuation (CV) bias terms:

$$\begin{aligned}\ln \Phi^g_{t-1,t} = &\underbrace{\frac{1}{\sigma^F_g-1}\ln\frac{\lambda^f_{gt,t-1}}{\lambda^f_{gt-1,t}}}_{\text{A: firm entry and exit}} + \underbrace{\frac{1}{\sigma^U_g-1}\sum_{f\in\Omega^F_{gt,t-1}}\omega^*_{fgt}\ln\frac{\lambda^u_{fgt,t-1}}{\lambda^u_{fgt-1,t}}}_{\text{B: goods (UPC) entry and exit}} \\ &+ \underbrace{\sum_{f\in\Omega^F_{gt,t-1}}\omega^*_{fgt}\ln\frac{\tilde{P}^*_{fgt}}{\tilde{P}^*_{fgt-1}}}_{\text{C: common goods price relative}} + \underbrace{\frac{1}{\sigma^U_g-1}\sum_{f\in\Omega^F_{gt,t-1}}\omega^*_{fgt}\frac{\tilde{S}^*_{fgt}}{\tilde{S}^*_{fgt-1}}}_{\text{C: consumer valuation bias}}.\end{aligned} \tag{3.39}$$

Equation 3.39 shows that the contribution of goods entry exit to the product group level price index is a weighted average of firm-level product entry and exit, with the weight being the Sato-Vartia weight, and adjusted for the elasticity of substitution between different goods. The product group level consumer valuation bias is the Sato-Vartia weighted firm-level consumer valuation bias, adjusted for the elasticity of substitution between different goods. The common goods price change is the Sato-Vartia weighted common goods price change at the firm level within a product group. Equation 3.39 also suggests that the between-firm elasticity $\sigma_g^F$ only affects the firm entry and exit term where there aren't firm-level demand shocks.

### 3.3.4 The Nested UPI with Firm-Level Demand Shocks

Introducing taste shocks at the firm level will give the same demand system as in Hottman et al. (2016):

$$C_{gt} = \left( \sum_{f \in \Omega_{gt}^F} (\varphi_{fgt} C_{fgt})^{\frac{\sigma_g^F - 1}{\sigma_g^F}} \right)^{\frac{\sigma_g^F}{\sigma_g^F - 1}} \quad C_{fgt} = \left( \sum_{u \in \Omega_{fgt}^U} (\varphi_{ufgt} C_{ufgt})^{\frac{\sigma_g^U - 1}{\sigma_g^U}} \right)^{\frac{\sigma_g^U}{\sigma_g^U - 1}}, \quad (3.40)$$

where $\varphi_{fgt}$ is firm appeal and $\varphi_{ufgt}$ is product appeal.

It can be shown that the product group level exact price index is

$$\ln \Phi_{t-1,t}^{g,full} = \frac{1}{\sigma_g^F - 1} \ln \frac{\lambda_{gt,t-1}^f}{\lambda_{gt-1,t}^f} + \frac{1}{N_{gt-1,t}^F} \sum_{f \in \Omega_{gt-1,t}^F} \ln \Phi_{t-1,t}^{fg} + \frac{1}{\sigma_g^F - 1} \frac{1}{N_{gt-1,t}^F} \sum_{f \in \Omega_{gt-1,t}^F} \ln \frac{S_{fgt}^*}{S_{fgt-1}^*}, \quad (3.41)$$

where $N_{gt-1,t}^F$ is the number of firms common in $t-1$ and $t$, $\Phi_{t-1,t}^{fg}$ is defined in equation 3.38, and $S_{fgt}^*$ and $S_{fgt-1}^*$ are common firm expenditure shares defined in equation 3.31.

Compared to equation 3.39, the formulation in equation 3.41 considers the expenditure share change between firms under relative firm-level taste shocks.

### 3.3.5 Inferring Relative Taste Shocks from Prices and Quantities

Using the estimated elasticity of substitution $\sigma$, we can invert the CES demand system to solve for time-varying demand shocks $\ln(\varphi_{kt}/\varphi_{kt-1})$.

The unit expenditure function for common goods is

$$P_t^* \equiv \left[ \sum_{k \in \Omega_{t,t-1}} \left( \frac{p_{kt}}{\varphi_{kt}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{3.42}$$

The expenditure share of a common good $k$ in period $(t-1, t)$ can be written as

$$s_{kt}^* = \frac{(p_{kt}/\varphi_{kt})^{1-\sigma}}{(P_t^*)^{1-\sigma}}, \tag{3.43}$$

hence

$$\varphi_{kt} = \frac{p_{kt}}{P_t^*} (s_{kt}^*)^{\frac{1}{\sigma-1}}. \tag{3.44}$$

Denote the geometric average of a variable $x$ over the common goods set by $\tilde{x}^*$. Taking geometric averages of both sides over the common goods set and dividing $\varphi_{kt}$ by $\tilde{\varphi}_t^*$, we have

$$\frac{\varphi_{kt}}{\tilde{\varphi}_t^*} = \frac{p_{kt}}{\tilde{P}_t^*} \left(\frac{s_{kt}^*}{\tilde{S}_t^*}\right)^{\frac{1}{\sigma-1}}, \quad \frac{\varphi_{kt-1}}{\tilde{\varphi}_{t-1}^*} = \frac{p_{kt-1}}{\tilde{P}_{t-1}^*} \left(\frac{s_{kt-1}^*}{\tilde{S}_{t-1}^*}\right)^{\frac{1}{\sigma-1}}. \tag{3.45}$$

Taking log on both side and applying the normalization assumption that the average product appeal of goods present in period $(t-1,t)$ is constant,[9] we have:

$$\ln \frac{\varphi_{kt}}{\varphi_{kt-1}} = \ln \frac{p_{kt}}{p_{kt-1}} - \ln \frac{\tilde{P}^*_t}{\tilde{P}^*_{t-1}} + \frac{1}{\sigma - 1}(\ln \frac{s^*_{kt}}{s^*_{kt-1}} - \ln \frac{\tilde{S}^*_t}{\tilde{S}^*_{t-1}}). \tag{3.46}$$

Equation 3.46 shows that relative demand shocks can be inferred from the observed price and quantity data for a given $\sigma$.

## 3.4 Estimating the Elasticity of Substitution

The computation of the exact price index relies on the estimates of *within* and *between* firm elasticities of substitution $\sigma^U_g$ and $\sigma^F_g$. I use the methodology in Hottman et al. (2016) (HRW), which is built upon Feenstra (1994) and Broda and Weinstein (2006). The elasticities are closely related to the estimates in Ehrlich et al. (2019b) and Ehrlich et al. (2020). I describe intuitions and key equations in the HRW estimation procedure in this section.

### 3.4.1 Estimating $\sigma^U_g$

The basic problem here is to obtain a demand and supply equation with information on prices and quantities only. The standard endogeneity problem exists where we do not know the slopes of the demand and supply equations that generated the observed data. Leontief (1929) suggests that the observed prices and quantities for a given good (UPC) can be represented by a hyperbola which is described by

[9] Redding and Weinstein (2020) assumes that the mean log-demand shocks across common goods is zero, i.e. $\ln(\tilde{\varphi}^*_t/\tilde{\varphi}^*_{t-1}) = 0$.

demand ($\sigma$) and supply ($\delta$) elasticities. The true elasticities maximize the likelihood function that satisfy the hyperbola.

The insight from Feenstra (1994) is that the panel structure of the data allows us to obtain a different hyperbola for each UPC as long as demand and supply shocks are not drawn from the same distribution as another UPC. Assuming that elasticities of supply and demand are the same for each UPC within a product group and that the relative variances of demand and supply shocks differ across UPCs, the hyperbolas do not overlap. Hence we can identify $\sigma$ and $\delta$ as the point at which the hyperbolas intersect.

Given the intuition above, the procedure can be written down formally. Double-differencing the log UPC expenditure share (3.22) over time and relative to the largest UPC within each firm gives:

$$\Delta^{ut} \ln S_{ufgt} = (1 - \sigma_g^U)\Delta^{ut} \ln P_{ufgt} + \nu_{ufgt}. \tag{3.47}$$

Similarly, let $\delta_g$ be the inverse supply elasticity of product group $g$, the double-differenced supply equation is given by [10]

$$\Delta^{ut} \ln P_{ufgt} = \frac{\delta_g}{1 + \delta_g}\Delta^{ut} \ln S_{ufgt} + \kappa_{ufgt}, \tag{3.48}$$

where $\nu_{ufgt}$ and $\kappa_{ufgt}$ are stochastic demand and supply residuals.

The first identifying assumption is that the double-differenced supply and

[10] In particular, we can write down the UPC pricing rule as $P_{ufgt} = \mu_{fgt}\gamma_{ufgt}$, with $\mu_{fgt}$ being the market up and $\gamma_{ufgt}$ being the marginal cost, and the cost function $A_{ufgt}(Y_{ufgt}) = a_{ufgt}(Y_{ufgt})^{1+\delta}$.

demand shocks are orthogonal. This follows standard approaches in the international trade and macroeconomics literature (Feenstra (1994), Broda and Weinstein (2006), Broda and Weinstein (2010)). The double-differencing procedure eliminates common firm-level shocks and time-invariant product-specific shocks and hence addresses the standard endogeneity concerns. The orthogonality condition gives a set of moment conditions:

$$G(\beta_g) = \mathbb{E}_t(\nu_{ufgt}\kappa_{ufgt}) = 0, \text{ with } \beta_g = \begin{pmatrix} \sigma_g^U \\ \delta_g \end{pmatrix}. \tag{3.49}$$

The second identifying assumption is that the double-differenced demand and supply shocks $\nu_{ufgt}$ and $\kappa_{ufgt}$ are heteroskedastic, i.e.

$$\frac{\chi^2_{\nu_u}}{\chi^2_{\nu_{u'}}} \neq \frac{\chi^2_{\kappa_u}}{\chi^2_{\kappa_{u'}}}, \tag{3.50}$$

with $\chi^2_x$ being the variance of $x$. This assumption ensures that the hyperbolas formed by different UPCs do not overlap.

For each product group, I stack all the UPC moment conditions together and the GMM estimator is given by

$$\hat{\beta}_g = \arg\min\{G^*(\beta_g)'WG^*(\beta_g)\}, \tag{3.51}$$

where $G^*(.)$ is the sample analog of $G(.)$, and $W$ is a positive definite weighting matrix. With consumer panel data from Nielsen, Hottman et al. (2016) weight the data for each UPC by the number of raw buyers for that UPC to reduce measurement errors. With scanner data, in a similar spirit, I weight the data for each UPC by the number of units sold to diminish the influence of unrepresentative prices.

Finally, the sample moment condition can be written as a function of observed price and quantities data and model parameters $\sigma_g^U$ and $\delta_g$ as follows (I omit subscripts $f, g$ for simplicity):

$$
\begin{aligned}
G^*(\beta_g) &= \frac{1}{T}\sum_t \omega_{ut}\kappa_{ut} \\
&= \frac{1}{T}\sum_t \left(\Delta^{\underline{ut}} \ln S_{ut}^U - (1-\sigma_g^U)\Delta^{\underline{ut}} \ln P_{ut}^U\right)\left(\Delta^{\underline{ut}} \ln P_{ut}^U - \frac{\delta_g}{1+\delta_g}\Delta^{\underline{ut}} \ln S_{ut}^U\right) \\
&= \frac{1}{T}\sum_t \Big((\sigma_g^U - 1)\big(\Delta^{\underline{ut}} \ln P_{ut}^U\big)^2 - \frac{\delta_g}{1+\delta_g}\big(\Delta^{\underline{ut}} \ln S_{ut}^U\big)^2 \qquad (3.52) \\
&- \frac{(\sigma_g^U - 2)\delta_g - 1}{1+\delta_g}\Delta^{\underline{ut}} \ln P_{ut}^U \Delta^{\underline{ut}} \ln S_{ut}^U\Big) \\
&= (\sigma_g^U - 1)\overline{\big(\Delta^{\underline{ut}} \ln P_{ut}^U\big)^2} - \frac{\delta_g}{1+\delta_g}\overline{\big(\Delta^{\underline{ut}} \ln S_{ut}^U\big)^2} - \frac{(\sigma_g^U - 2)\delta_g - 1}{1+\delta_g}\overline{\Delta^{\underline{ut}} \ln P_{ut}^U \Delta^{\underline{ut}} \ln S_{ut}^U} \\
&= (\sigma_g^U - 1)\bar{y}2(u) - \frac{\delta_g}{1+\delta_g}\bar{z}2(u) - \frac{(\sigma_g^U - 2)\delta_g - 1}{1+\delta_g}\bar{z}1(u), \qquad (3.53)
\end{aligned}
$$

where $\beta_g = (\sigma_g^U \delta_g)'$, $\overline{X_{ut}} = \frac{1}{T}\sum_t X_{ut}$.

### 3.4.2 Estimating $\sigma_g^F$

Given the estimate of $\sigma_g^U$, I can compute firm price indices based on 3.38 to get $P_{fgt}$. Double-differencing the firm expenditure share over time and relative to the largest firm within each product group, we have:

$$\Delta^{\underline{f},t} \ln S_{fgt} = (1 - \sigma_g^F)\Delta^{\underline{f},t} \ln P_{fgt} + \epsilon_{fgt}. \tag{3.54}$$

Estimating equation 3.54 using OLS could be problematic because changes in firm price indexes can be correlated with changes in firm appeal. To find an instrument for $\ln P_{fgt}$, taking log of 3.20 and utilizing the fact that

$$\frac{S_{ufgt}}{\tilde{S}_{fgt}} = \left(\frac{P_{ufgt}/\varphi_{ufgt}}{\tilde{P}_{fgt}/\tilde{\varphi}_{fgt}}\right)^{1-\sigma_g^U}, \tag{3.55}$$

and the normalization assumption $\tilde{\varphi}_{fgt} = 1$, we have

$$\ln P_{fgt}^F = \ln \tilde{P}_{fgt}^F + \frac{1}{1 - \sigma_g^U} \ln \sum_{u \in \Omega_{fgt}} \left(\frac{S_{ufgt}}{\tilde{S}_{fgt}}\right). \tag{3.56}$$

The last term in equation 3.56 measures the dispersion of the expenditure shares. The structure of the model implies that the dispersion of the UPC expenditure share only affects the share of the firm through the firm price index $P_{fgt}$. Double-differencing equation 3.56 over time and relative to the largest firm within each product-group, we have:

$$\Delta^{\underline{f},t} \ln P_{fgt}^F = \Delta^{\underline{f},t} \ln \tilde{P}_{fgt}^F + \frac{1}{1 - \sigma_g^U} \Delta^{\underline{f},t} \ln \sum_{u \in \Omega_{fgt}} \left(\frac{S_{ufgt}}{\tilde{S}_{fgt}}\right). \tag{3.57}$$

The second term on the right hand side containing the UPC expenditure shares is a suitable instrument for the double differenced firm price index in equation 3.54. [11]

## 3.5 Estimated Elasticities of Substitution

Table 3.3 summarizes the estimated "within" ($\sigma^U$) and "between" ($\sigma^F$) firm elasticities for the 106 product groups, as well as a flat $\sigma^U$ using the Feenstra (1994) procedure. Note that the difference in estimating procedures for the flat $\sigma^U$ and the within $\sigma^U$ lies in how we perform the double-differencing operation. When estimating within $\sigma^U$, I double difference along time and relative to the largest firm within each product-group; while when estimating flat $\sigma^U$, I double-difference along time and relative to the geometric average of the variable in the product group.

The median estimate of $\sigma^U$ is 6.65, and $\sigma^U$ ranges from 3.42 at the 5th percentile to 18.12 at the 95th percentile. $\sigma^F$ ranges from 1.24 at the 5th percentile to 5.56 at the 95th percentile, with the median being 2.29. The fact that $\sigma^U$ is greater than $\sigma^F$ suggests that goods within a firm is more substitutable than goods between firms.

This set of estimates is comparable to the estimates obtained in the literature. Using the Consumer Panel data from Nielsen, Hottman et al. (2016) estimate that the median of $\sigma^U$ to be 6.9 with the 95th and 5th percentile being 17.6 and 4.7. The median of $\sigma^F$ from HRW is 3.9, with the the 95th and 5th percentile being 8.5 and 2.3. My estimated $\sigma^F$ from the Retail Scanner data is on average lower than HRW's estimates from the Consumer Panel. This may suggest that goods supplied

[11] Ehrlich et al. (2020) consider improving the HRW between firm estimator by using common goods expenditure shares as an instrument to further eliminate the concern that $\epsilon_{fgt}$ might be correlated with the number of goods.

| Percentile | $\sigma^U$ | $\sigma^F$ | $\sigma^{U,Flat}$ |
|---|---|---|---|
| Min | 2.80 | 1.06 | 3.07 |
| 5% | 3.42 | 1.24 | 3.65 |
| 10% | 4.12 | 1.39 | 4.24 |
| 25% | 4.81 | 1.76 | 5.35 |
| 50% | 6.65 | 2.29 | 8.06 |
| 75% | 9.48 | 3.05 | 11.55 |
| 90% | 13.00 | 4.05 | 16.14 |
| 95% | 18.12 | 5.56 | 18.49 |
| Max | 38.83 | 15.93 | 28.90 |

Table 3.3: Distribution of the estimated elasticities across 106 Nielsen RMS product groups

by smaller firms are harder to substitute between firms. For the flat $\sigma^U$, Redding and Weinstein (2020) report a median of 6.48 with the 95th and 5th percentile being 8.5 and 5.1 using data from Nielsen Consumer Panel. The estimates are also consistent with Ehrlich et al. (2019b) where the median estimate for the flat $\sigma$ is 8 using the scanner data.

## 3.6 The Measured Cost of Living

I now turn to the price indices estimated from the Nielsen Retail Scanner (RMS) data. I construct the following five indices: (1) Laspeyres, (2) Sato-Vartia, (3) Sato-Vartia with Feenstra adjustment for product entry and exit, (4) flat UPI, and (5) nested UPI. I calculate price indices for each product group and the aggregate index is a Divisia-weighted index of the product group level price indices. The nested UPI in this section is the baseline case defined in equation 3.36 through 3.38.

### 3.6.1 Aggregate Price Indices

I first look at the aggregate price indices for the consumer goods sector represented by the 106 Nielsen RMS product groups from 2006 to 2015. Figure 3.1 plots the chained indices. The Laspeyres index was largely flat during this time period, and the Sato-Vartia index declined by 12 percent from 2006 to 2015. Product entry and exit, represented by the Feenstra adjustment term to the Sato-Vartia index contributed an additional 8-percent decline to the Sato-Vartia index. The flat UPI declined by 26 percent and the nested UPI declined by 11 percent.

Figure 3.2 shows the corresponding annual growth rate for each index. The average annual inflation measured by the Laspeyres index is 0.03%, by the Sato-Vartia index is $-1.2\%$, and by the Feenstra-adjusted Sato-Vartia index is $-2.1\%$. The flat UPI shows an annual inflation of $-3.1\%$, and the nested UPI shows an annual inflation rate of of $-1.0\%$. The nested UPI lies above the flat UPI, suggesting that the between-goods substitution is better captured in a nested structure. In other words, goods within a firm are more substitutable than goods between firms. In terms of the cyclicality, all five indices exhibit similar cyclical behaviors, with the nested UPI shows a more significant decline in 2009 than the other four.

The nested UPI exhibits a downward adjustment to the traditional CPI measured by a Laspeyres index. To study what leads to this negative adjustment, I breakdown the nested UPI into four components: the firm entry and exit adjustment, the product entry and exit adjustment, common goods price relative and the consumer valuation bias adjustment according to equation 3.39.

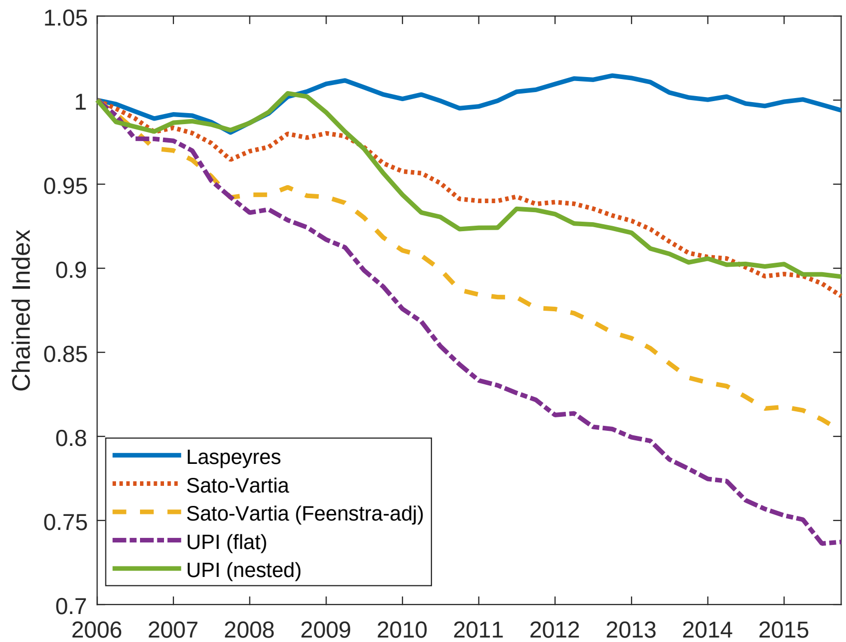


Figure 3.1: Aggregate price indices. This figure plots the chained price indices for all Nielsen Retail Scanner product groups, with indices at 2006 Q1 normalized to 1. The solid blue line is the Laspeyres index. The red dotted line is the Sato-Vartia index. The yellow dashed line is the Sato-Vartia index with Feenstra adjustment term for product entry and exit. The purple dash-dotted line is the flat UPI and the solid green line is the nested UPI.

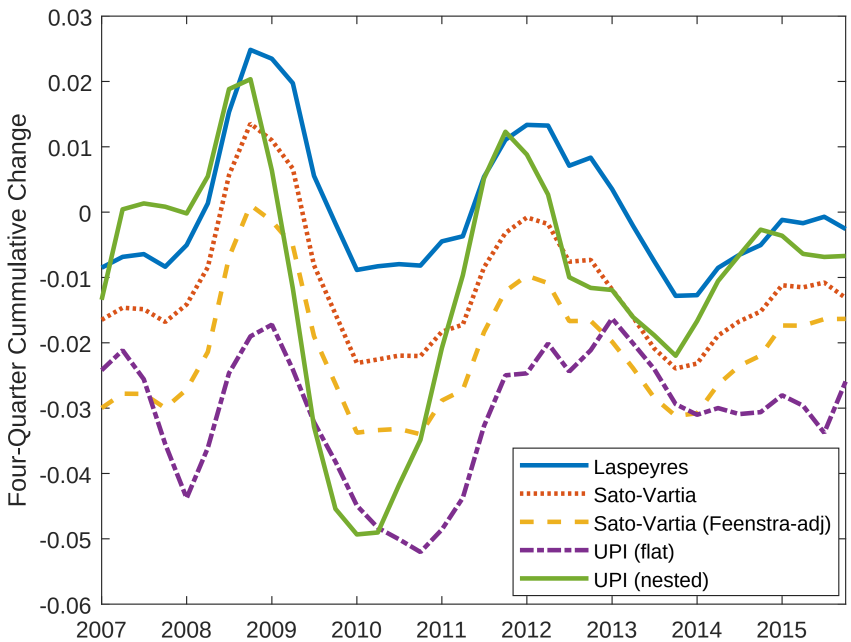


Figure 3.2: Aggregate price indices in annual growth rate. This figure plots the four-quarter cummulative percentage change of the price indices. The solid blue line is the Laspeyres index. The red dotted line is the Sato-Vartia index. The yellow dashed line is the Sato-Vartia index with the Feenstra adjustment term for product entry and exit. The purple dash-dotted line is the flat UPI and solid green line is the nested UPI.

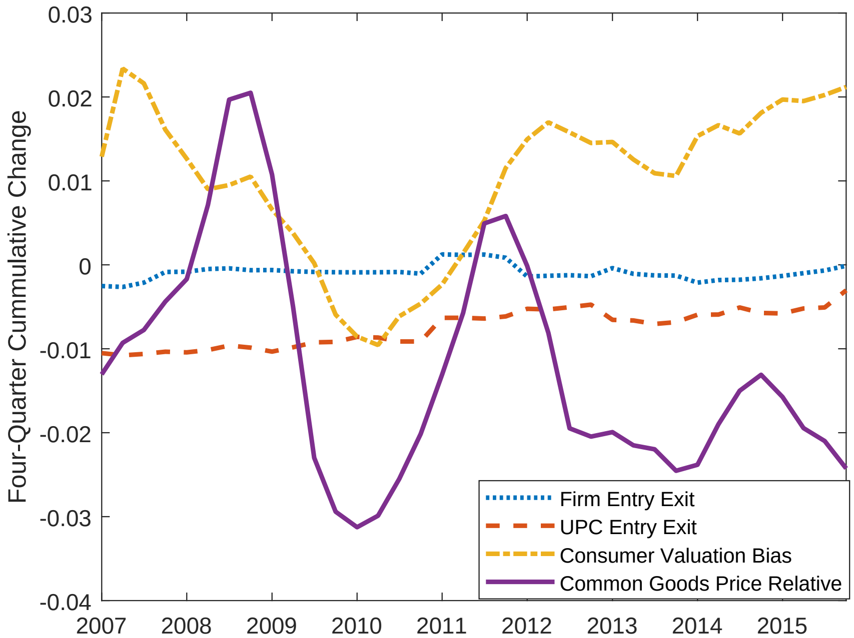


Figure 3.3: This figure shows the decomposition of the annual inflation rate into four components: firm entry and exit, product entry and exit, consumer valuation bias and common goods price relative (Jevons index). The blue dotted line is the firm entry and exit adjustment component; the red dashed line is the product entry exit component, the yellow dash-dotted line is the consumer valuation bias and the solid purple line the common goods price relative.

Figure 3.3 shows the result. The firm entry and exit adjustment is largely flat, suggesting that entering firms don't necessarily have higher quality goods. This may due to the fact that it takes time for a new firm to roll out its products, or it takes time for consumers to appreciate a new product. Product turnover, measured by the UPC entry and exit component, contributes 0.8% annual decline in the cost of living, suggesting that entering goods typically have better quality than exiting goods. Common goods price relative (Jevons index) is an unweighted average of price changes of goods within firms. This component mimic the fluctuation of the Laspeyres index and averaged at $-1.2\%$. Negative common goods price relative suggests that goods in the economy has declined in nominal price over time. The gap between the common goods price relative and the Laspeyres index implies that prices of small share goods have declined more than prices of large share goods.

The consumer valuation bias component has an average of 1.0%. It declined from 2007 to 2010 and increased afterwards. This term is calculated as an unweighted average of the expenditure share relatives and is a variant of the Theil index of dispersion. When the dispersion of expenditure shares increases overtime, the consumer valuation bias is negative; when the dispersion shrinks, the bias turns positive. In particular, the dispersion of expenditure shares will rise when "winning" (large share) goods keep gaining market share, and "loosing" (small share) goods keep loosing market share. In contrast, when small share goods win market share over large share goods, the consumer valuation will be positive.

### 3.6.2 The Common Goods Rule

One feature of the consumer valuation bias is that it is an unweighted average of common goods expenditure share changes from $t-1$ to $t$. Hence any given dollar amount of expenditure loss (gain) of a small share good to (from) a large share good will result in a negative (positive) bias, as this dollar amount change will incur a large decline (increase) in the expenditure share of the small share good but only a small increase (decline) in the expenditure share of the large share good. The sensitivity of the consumer valuation bias to small share goods and the presence of a large left tail in the Nielsen Retail Scanner data may give rise to a significant negative bias in the measured UPI. In particular, if small shares are related to the geographic roll-out of a good, or to the slow death process of goods, the measured UPI will not correctly reflect changes in the cost of living for a national representative consumer.

I apply a common goods rule (CGR) to the data to account for this small share issue (this CGR will be further analyzed in Ehrlich et al. (2020)). This common goods rule says that in order for a good to be counted as a common good between $t-1$ to $t$, it needs to have positive sales in both time $t$ and time $t-1$ and its sales must be above a certain threshold. Otherwise, this good will be classified as an entering or exiting good. This rule is similar to the longevity rule used in Redding and Weinstein (2020) where they require a good to be present in the sample for a total of 6 years or more in order to be counted as a common good. The drawback of their method is twofold. First, the 6-year window is too long when considering the life cycle of products in the innovation intensive sector, such as electronics, and second, it is a "backward looking" method and can not be implemented in real time.

The threshold in the empirical analysis is inferred from the Nielsen Consumer Panel data (HMS). In particular, I construct a flat UPI using HMS and define the threshold to be the $x^{th}$ percentile of the sales distribution calculated using UPC sales information in the 4 quarters preceding $t$ for each product group at any time $t$.[12] I vary $x$ until the calculated UPI matches the UPI in Redding and Weinstein (2020). This procedure gives an $x$ of 5. I then calculate the corresponding dollar amount of sales of the $5^{th}$ percentile of the sales distribution at any $t$ for every product group. A time series of the implied threshold for each product group is then applied to the Retail Scanner Data. The implied threshold is equivalent to the $30^{th}$ to $50^{th}$ percentile of the sales distribution in the RMS.

In a back-of-the-envelop calculation, I shown that small shares (sales) correspond to the entry and exit process, i.e. entering and exiting goods tend to have smaller sales and expenditure shares. In this sense, the CGR used in my analysis can be regarded to as a real-time version of the longevity rule in Redding and Weinstein (2020).

### 3.6.3 Food vs. Non-Food

Nielsen retail scanner covers grocery and mass merchandiser stores sales and the goods can be grouped into food and non-food categories. Ehrlich et al. (2019a) compare the BLS and Nielsen data sources in detail and show that the Laspeyres

[12] I choose to include sales information for the past 4 quarters since it gives more data to compute a reliable distribution and it will also takes into account the seasonality in sales.

and the Feenstra indices constructed from the scanner data follow closely the BLS CPI food and non-food series and that the traditional price indices are missing substantial quality adjustments. In this section, I compare the nested UPI to the traditional price indices in the food and non-food sector separately, and explore differences in the food and non-food sector by looking at the components of the nested UPI.

Figure 3.4 shows the results. In the food sector, as shown in Panel (A1) of Figure 3.4, tradition CPI (Laspeyres) implies a rise in the cost of living by 32% from 2006 to 2015. Increases in the cost of living measured by the Sato-Vartia and the Feenstra-adjusted Sato-Vartia indices are at 9% and 6% respectively. The flat UPI implies an increase of 51%. The nested UPI implies an increase in the cost of living by 20% from 2006 to 2015. In terms of annual inflation rates, as shown in Panel (A2) of Figure 3.4, the Laspeyres index gives an annual inflation of 3.0%. The inflation rates given by the Sato-Vartia index and the Feenstra-adjusted Sato-Vartia index are 1.0% and 0.7% respectively. The flat UPI implies an annual inflation of 4.4% and the nested UPI implies an inflation rate of 2.0%.

In the non-food sector, as shown in Panel (B1) of Figure 3.4, the measured cost of living declined from 2006 to 2015 in all indices except for the tradition CPI (Laspeyres), which remained flat during this period. The Sato-Vartia and Feenstra-adjusted Sato-Vartia index imply a drop in the cost of living by 12% and 20% respectively. The flat UPI implies a drop in the cost of living by 26% and the nested UPI implies a drop by 40% from 2006 to 2015. In terms of annual rates, as shown in Panel (B2), the Laspeyres index gives an average annual inflation of 0.0%, the

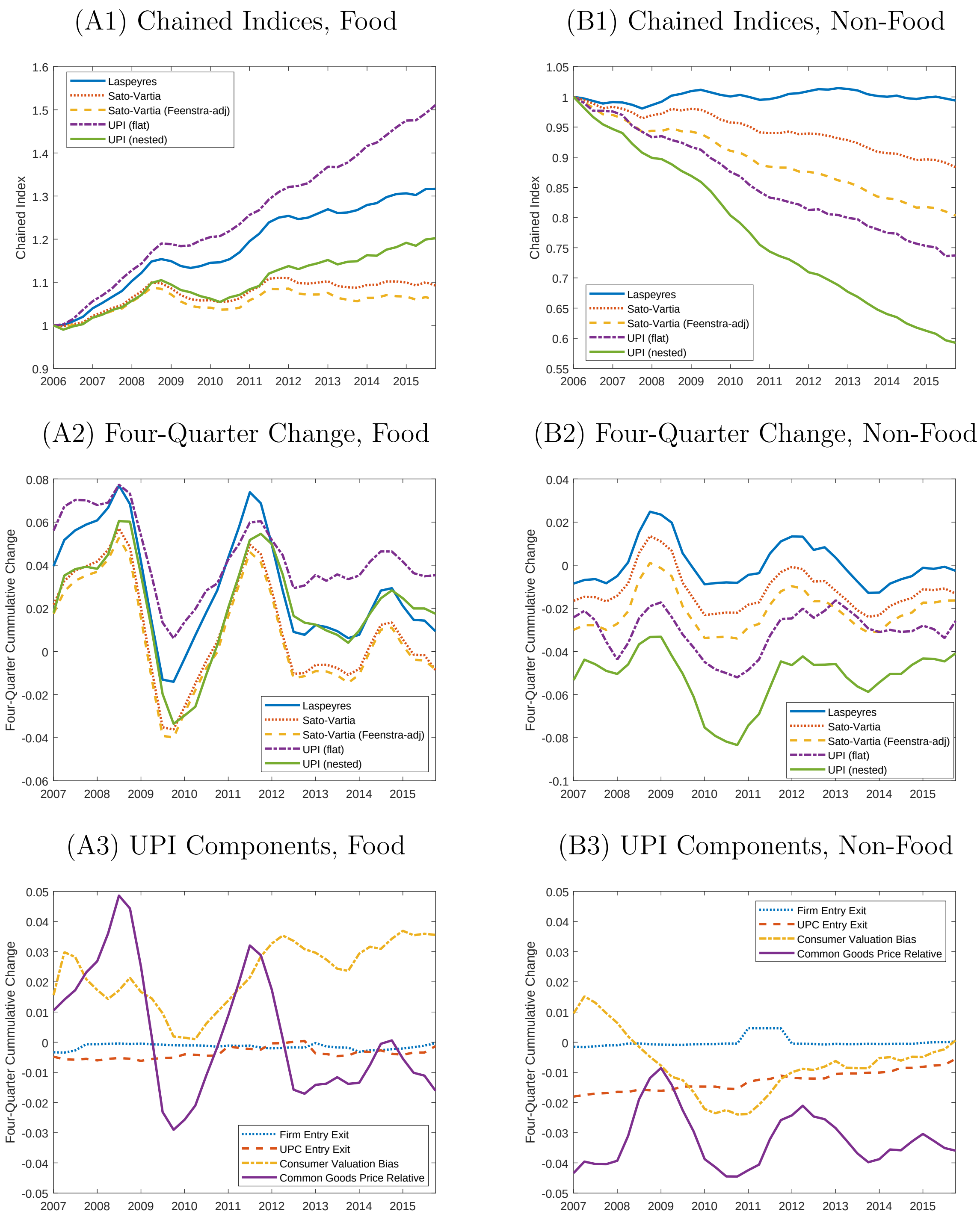


Figure 3.4: Food vs. non-food price indices and the UPI decomposition. Panel (A1) to (A3) are for the food sector and (B1)-(B3) for non-food. (A1) and (B1) plot the chained indices with 2006 q1 normalized to 1. (A2) and (B2) plot the four-quarter cumulative changes. (A3) and (B3) plot the components of the nested UPI.

Sato-Vartia and Feenstra-adjusted Sato-Vartia indices give annual inflation rates of $-1.2\%$ and $-2.1\%$ respectively. The flat UPI implies an annual inflation of $-5.2\%$ and the nested UPI implies an annual rate of $-3.1\%$.

Panel (A3) and (B3) show the four components of the nested UPI. UPC entry and exit are more quality enhancing in the non-food sector, with and average annual rate of $-1.3\%$. While in the food sector, the rate is only -0.4%. This UPC entry and exit adjustment term suggests that there are more innovation in the non-food sector compared to food. Firm entry and exit adjustments are flat in both sectors. The Jevons index is more negative in the non-food sector, suggesting that firms lower product prices over time. This may also be a result of innovation, where successful innovations help firms to lower marginal costs and hence reduce prices to gain market share. Meanwhile, the Jevons index demonstrates similar cyclical fluctuations in both food and non-food sectors. The consumer valuation adjustment is positive in the food sector but negative in the non-food sector, suggesting that consumer expenditure in the food sector have become less disperse over time while is has become more disperse in the non-food sector.

The gap between the flat and nested UPI is mainly attributable to the consumer valuation bias term which is higher in the flat UPI than in the nested UPI for both sectors. The CV term in the flat UPI measures the effect of relative taste shocks across all goods in a sector, while the CV term in the nested UPI measures the average effect of relative taste shocks between goods within a firm in the sector. The high CV in the flat UPI suggests that within a product group, small share goods tend to receive positive taste shocks relative to large share goods, so that expenditure shares have become *less dispersed* over time. The low CV in the nested UPI says that within a firm, large share goods tend to receive positive demand shocks and grow even larger, so the dispersion of expenditure shares of goods within a firm has become *more dispersed* over time.

### 3.6.4 Empirical Test of Proposition 1

Now I test the predictions of Proposition 1 and Redding and Weinstein (2020) based on the empirical strategy described in 3.3.2.2. In the first step, I estimate the correlation between initial period market shares and relative taste shocks for each product group ($a_g$), and the correlation between changes in market shares and relative taste shocks for each product group ($b_g$). In the second step, I estimate the relationship between $a_g$, $b_g$ and the consumer valuation bias.

If Proposition 1 holds in the data, we should expect to see that the slope between $a_g$ and $CV_g$ to be negative, i.e. the more correlated the initial size of a product and relative taste shocks, the more negative the consumer valuation bias term. If RW's argument holds in the data, we should expect to see that the more correlated the relative taste shocks and the change in market share, the more negative the consumer valuation bias.

Figure 3.5 shows the results. Panel (A) shows that indeed, there is negative relationship between consumer valuation bias and $a_g$, the correlation between initial market share and relative taste shocks. The estimated coefficient $\beta$ (equation 3.16) is $-0.14$ with a standard error of 0.015.[13] On the other hand, there isn't a negative

[13]The estimation in the second step may be subject to generalized regressors problem where the

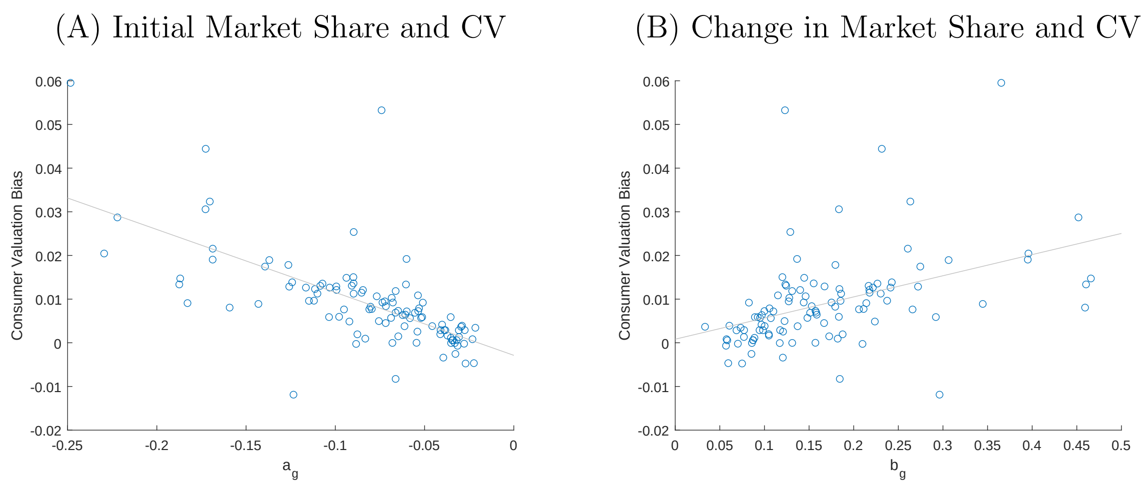


Figure 3.5: Test predictions of Proposition 1. Panel (A) plots the relationship between the estimated correlation between the initial market share of products and relative taste shocks and the size of the consumer valuation bias across all product groups. Panel (B) plots the relationship between the estimated correlation between changes in market share of products and relative taste shocks and the size of the consumer valuation bias across all product groups.

correlation between consumer valuation bias $b_g$, and the correlation between change in market share and relative taste shocks. Panel (B) shows that consumer valuation bias and $b_g$ is instead positively correlated. The estimated coefficient $\gamma$ (equation 3.17) is 0.05 with a standard error of 0.01. Therefore, the RW's argument doesn't hold in the data.

In sum, Proposition 1 holds in the data and suggests that the sign of the consumer valuation bias is determined by the correlation between the initial period size of a product and relative taste shocks. Therefore, taste shocks to products of large firms may have different price impact from taste shocks to products of small firms.[14]

---

$a_g$ it self is a random variable. Methods such as bootstrapping can be used to get a more precise standard error on $\beta$.

[14]Hottman et al. (2016) shows that products in large firms tend to be large in size.

### 3.6.5 Contribution of Large Firms to the Cost of Living

An advantage of the nested structure is that it allows me to analyze firms' contribution to the cost of living systematically. In particular, it allows me to study how firms affect the components of the nested UPI. I perform counterfactual experiments where I set firm price indices of top 5, 10, and 20 firms (in terms of expenditure shares) in each product group to be constant over time, and evaluate what the aggregate UPI would be under these scenarios. I also re-normalize the Sato-Vartia weights so that the change in the prices indices will not be biased mechanically towards zero. Such experiments are equivalent to dropping corresponding firms from the sample. In the nested CES setting, dropping firms effectively shuts down substitution of goods between firms while the substitution within firms is unaffected.[15]

#### 3.6.5.1 The Aggregate Consumer Goods Sector

I first look at the aggregate consumer goods sector. Figure 3.6 shows the result. Without top 5 firms in each product group, the cumulative change in the cost of living would have increased by 14% from 2006 to 2015. Further excluding top 5 to 10 firms would have further increased the cost of living by 3%. Excluding top 10 to 20 firms will not change the aggregate index further. In terms of annual inflation rate, dropping top 5 firms would have increased the inflation rate from -1% to 0.4%. The annual inflation rate would have increased to 0.7% when dropping top

[15] In contrast, in the flat UPI structure, dropping firms will affect the substitution both within and between firms.

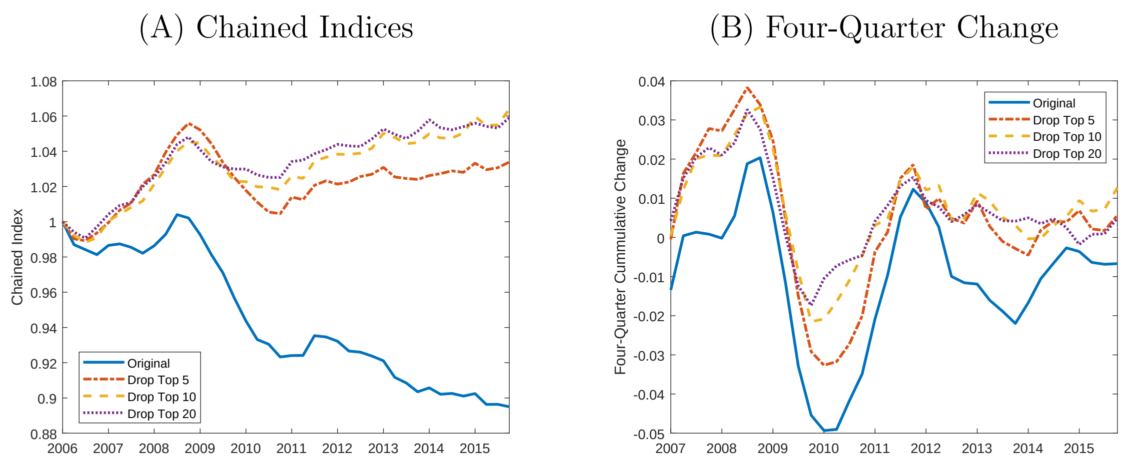


Figure 3.6: UPI counterfactuals for the aggregate consumer goods sector. This figure shows the nested UPI from the counterfactual experiments where I drop top 5, 10, 20 firms in terms of expenditure shares in each product group. Panel (A) plots the chained indices and Panel (B) plots four-quarter cumulative growth rates.

10 firms and stays at 0.7% when further excluding firms between top 10 and top 20.

To understand what drives the change in the measured inflation, we can look at the components of the nested UPI. Panel (A) of Figure 3.7 shows that firm creation and destruction is productivity enhancing. Moreover, smaller firms have more active entry and exit dynamics and contribute to the decline of the cost of living. Panel (B) shows the contribution of product (UPC) entry and exit to the aggregate cost of living. Without the top 20 firms in each product group, the contribution of product entry and exit declined by 3 percent. This suggest that the product creative destruction is more active in large firms than in small firms, even though the difference is also significant (0.3% on an annual basis). Panel (C) shows that the common good price relative declines more in small firms than in large

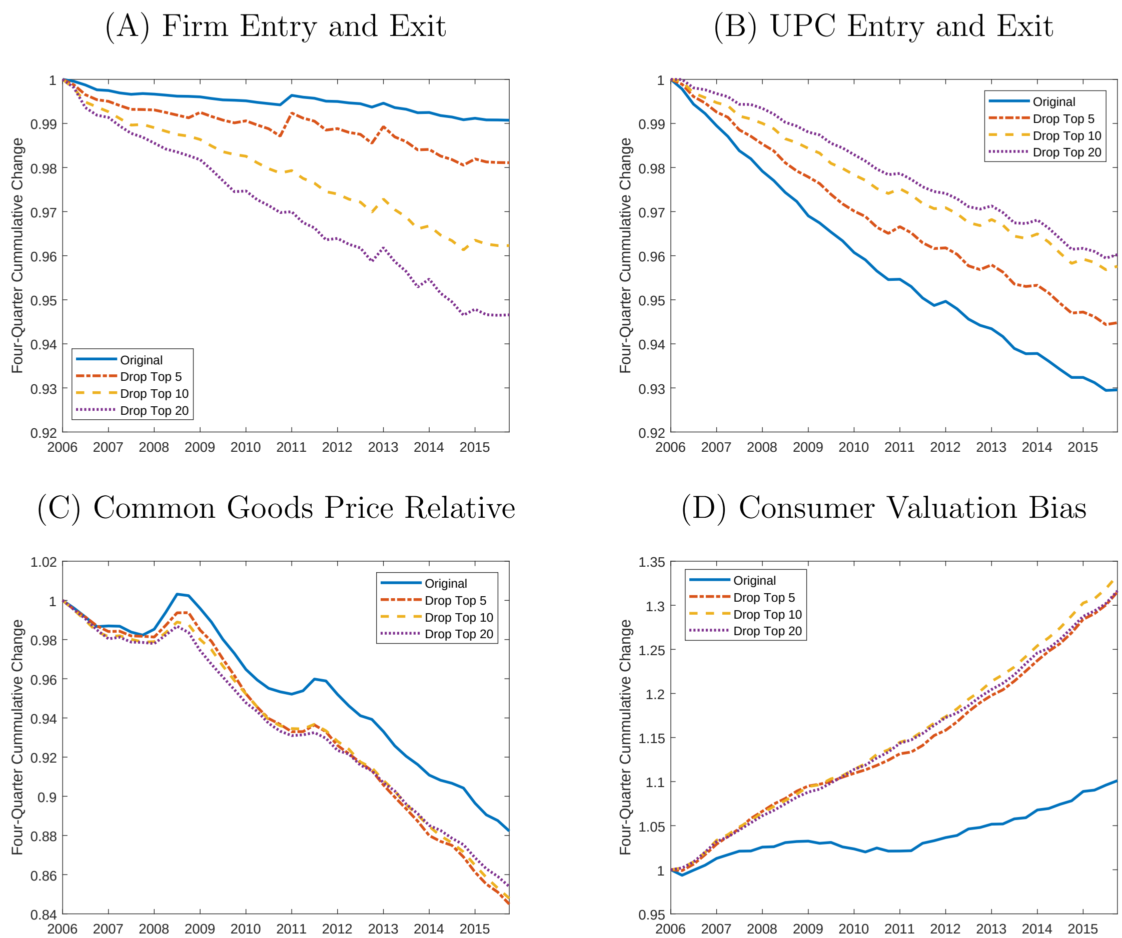


Figure 3.7: This figure shows components of UPI in the counterfactual experiments where we drop top 5, 10, 20 firms in terms of expenditure shares in each product group. Panel (A) plots the firm entry and exit adjustment component, Panel (B) plots the UPC entry and exit adjustment component; Panel (C) plots the common goods price relative (Jevons index) and Panel (D) plots the consumer valuation bias term.

firms, suggesting that smaller firms are more aggressive in reducing product prices. Moreover, the difference mainly happens between the top 5 firms and the rest.

The consumer valuation bias exhibits the most significant change with and without large firms, as shown in Panel (D) of Figure 3.7. If top 5 firms in each of the product groups were excluded, the consumer valuation bias would have driven up the cost of living by 32% from 2006 to 2015. Further excluding firms between top 5 to top 20 will not change the consumer valuation bias further. The consumer valuation bias term in the nested UPI measures the average dispersion of UPC expenditure shares within firms. Results in Panel (D) implies that expenditure share within small firms have become less dispersed over time. In other words, small share products in small firms received positive relative taste shocks over time.

#### 3.6.5.2 Electronics vs. Snacks

The results above are for the aggregate consumer goods sector represented by all Nielsen product groups. There are also important heterogeneity across different product groups. In particular, I compare two product groups: electronics and snacks. Electronics is an innovation intensive product group and snacks belongs to the food sector which is regarded as less innovation intensive.[16]

Figure 3.8 shows the results. Electronics exhibits deeper deflation compared to snacks. The cost of living dropped by 80% in electronics between 2006 and 2015, with an annual inflation rate of -15%. In snacks, in contrast, the cost of living only

[16] Admittedly, the coverage of electronics in Nielsen is limited but it does include products such as computer software, video games, printers, and etc.

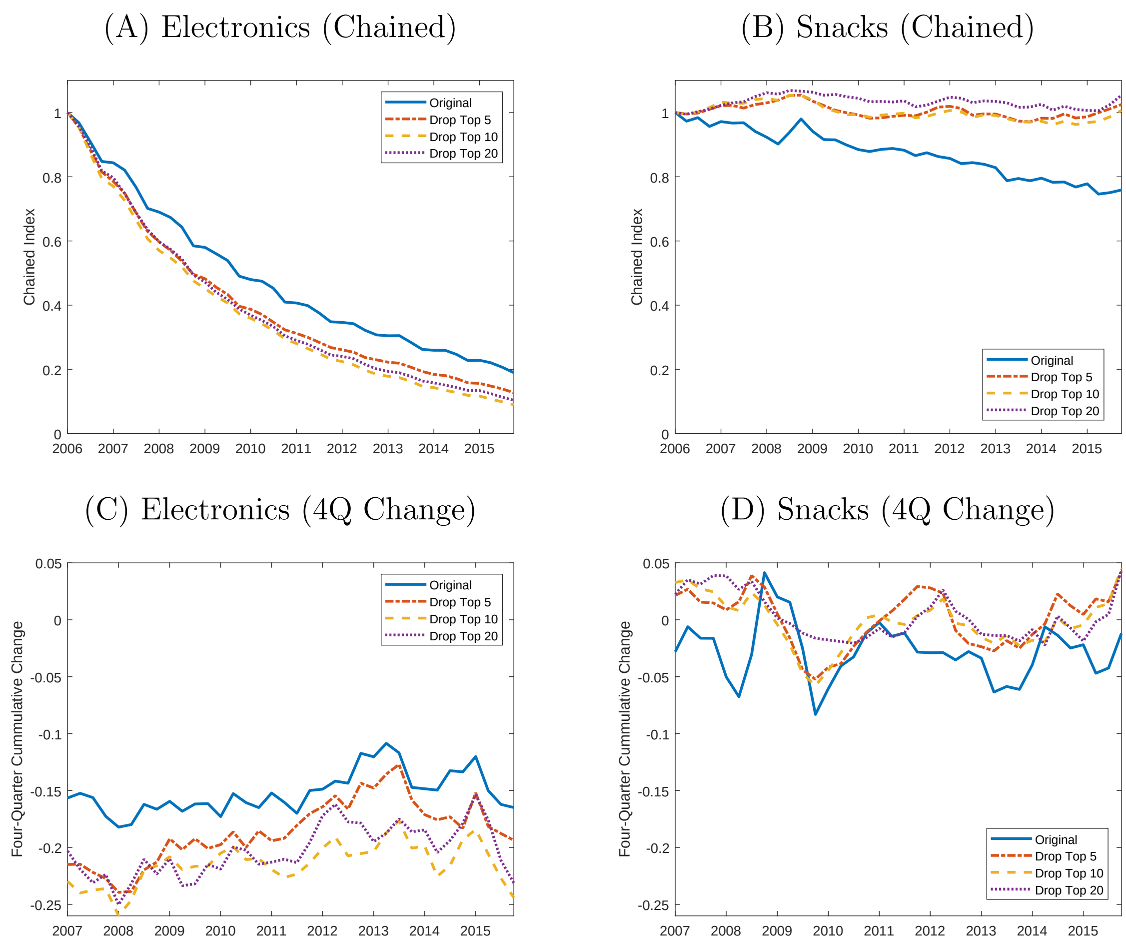


Figure 3.8: This figure shows UPI counterfactuals for electronics and snacks where I drop top 5, 10, 20 firms in terms of expenditure shares in each product group. Panel (A) and (B) plot the chained indices for electronics and snacks. Panel (C) and (D) plot the four-quarter cumulative change for the two product groups.

declined by 20% during the same period with an annual inflation rate of -2.8%. The role of large firms are also different in the two product groups. Excluding the top 5 firms in electronics, the annual deflation rate deepened from 15% to 19%. The rate stays at 20% when further excluding top 5 to top 20 firms. While in snacks, the cost of living in this sector increases without the large firms. Annual inflation rate increases from $-2.8\%$ to 0.2% when excluding the top 20 firms.

To understand what drives the changes, Figure 3.9 compares the UPI components in the experiments for these two product groups. A vary noticeable feature for electronics, as shown in Panel (A2), is the significant contribution of the UPC entry and exit component which averaged at -21%, indicating a very intensive creative destruction process in this sector. Moreover, the product-level creative destruction is more significant in large firms. Excluding the top 5 firms, the UPC entry and exit component shrinks to -13% and the rate further shrinks to -6% when excluding the top 20 firms. In contrast, in snacks, there is almost no contribution to the cost of living from product entry and exit.

The second most noticeable component for electronics is the consumer valuation bias term, as shown in Panel (A4) of Figure 3.9. It is positive in the baseline case and declines as I drop the top firms. This suggests that the within firm expenditure dispersion is lower in large firms. Given Proposition 1, this result suggests that within large firms, relative taste shocks favor small share products. The consumer valuation bias term averaged around 0 in snacks and remains largely unchanged in the counterfactual experiments, as shown in Panel (B4).

The common goods price relative is negative in electronics while it fluctuates

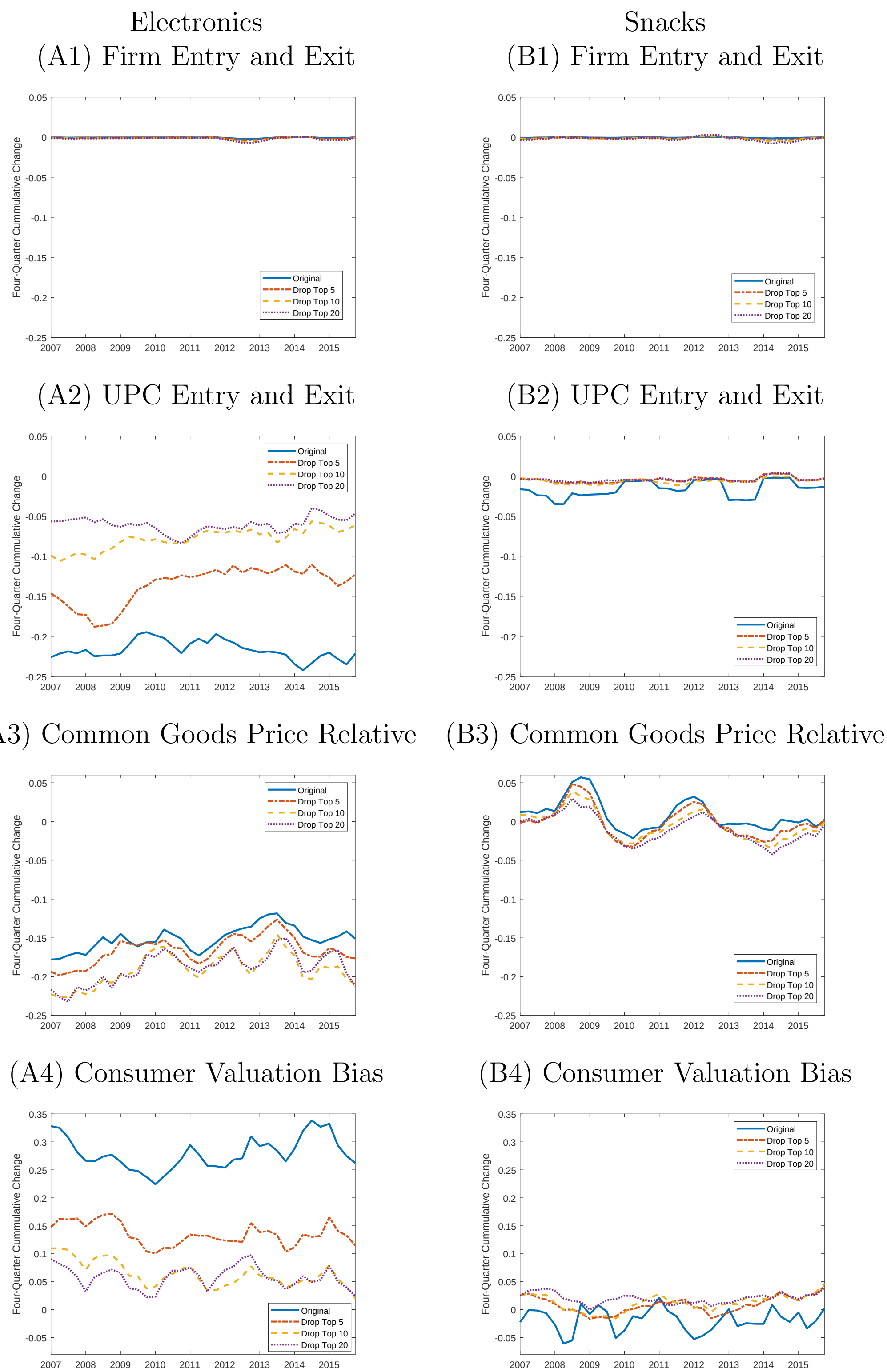


Figure 3.9: Components of UPI under the counterfactual experiments for electronics and snacks product groups. Panels (A1)-(A4) are for electronics. Panels (B1)-(B4) are for snacks.

around 0 for snacks, as shown in Panel (A3) and (B3). The negative Jevon index suggests that in the electronics sector, firms lower prices over time. This can be a result of cost savings through innovation or increased market competition. Moreover, Panal (A3) also suggusts that small firms in electronics tend to lower price more aggressively than large firms.

Finally, the firm entry and exit component is largely flat in both sectors.

### 3.6.6 Nested UPI with Firm-Level Taste Shocks

Finally, I turn to the comparison between the baseline UPI (defined in equation 3.36 through 3.38) and the UPI with firm-level taste shocks (full UPI) defined in equation 3.41. Adding firm-level taste shocks allows me to look at the effect of relative taste shocks between firms.

Figure 3.10 shows the comparison between the Laspeyres index, the flat UPI, the baseline nested UPI and the full UPI for the aggregate consumer goods sector. We can see that the full UPI exhibits significant volatility even though the average inflation rate is the same as the rate implied by the baseline nested UPI.

The volatility is more noticeable when we look at quarterly series, as shown in Panel (C). Even though the mean quarterly log change is 0% for both the baseline and the full UPI, the volatility of the full UPI is 5 times larger and exhibits some seasonal patterns. This volatility is fully driven by the consumer valuation bias between firms, as shown by the green dotted line in Panel (C), suggesting that there exist seasonal relative taste shocks between firms.

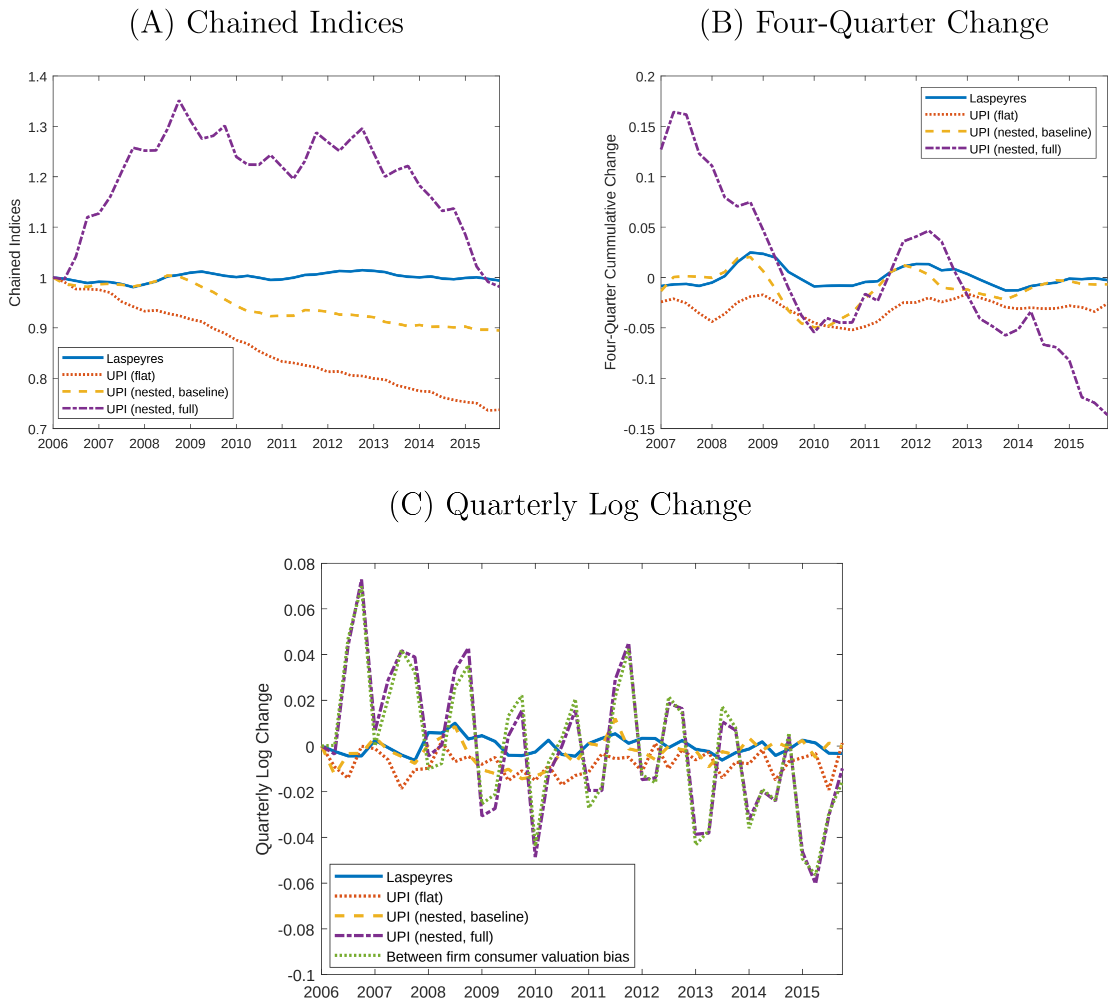


Figure 3.10: This figure compares the full UPI to other indices. Panel (A) shows the chained indices and Panel (B) shows the four-quarter cumulative change. The solid blue line is the Laspeyres index; the red dotted line is the flat UPI, the yellow dash line is the nested UPI and the purple dash-dot line is the nested UPI with firm-level taste shocks. The green dotted line is the between firm consumer valuation bias term of the full UPI

## 3.7 Conclusion

In this chapter, I study the measurement of the exact cost of living at the presence of product and firm-level creative destruction and relative taste shocks and the contribution to the living cost by firms of different sizes by utilizing the UPI framework developed by Redding and Weinstein (2020). I show analytically in a two-good economy that under the UPI framework, the correlation between firm size and relative taste shocks determines the direction of the consumer valuation bias. This is different from RW's argument that relative taste shocks will always introduce a negative consumer valuation bias to the traditional Sato-Vartia index. I test this proposition using the inferred taste shocks from the UPI framework and the estimated elasticity of substitutions. The results favor my proposition while doesn't favor the argument from RW. Such results also suggest that innovations may not always reduce the cost of living for consumers. This happens when innovations improve the relative appeal of small share goods and hence reduce the the dispersion among substitutable goods.

I construct a nested UPI index which considers multi-product firms and the different substitutability within and between firms. Using the Nielsen Retail Scanner dataset, I show that the annual inflation rate measured by the nested UPI is -1%, 1% lower than the inflation measured by the Laspeyres index. For the food and non-food sector, the measured inflation rate by UPI is 2.0% and -3.1% respectively, compared to 3.0% and 0.0% measured by the Laspeyres index.

Through counterfactual experiments, I show that large firms help drive down the cost of living in aggregate by lowering the consumer valuation bias and their roles can be different across sectors. In the innovation intensive sector (electronics), large firms engage in more active product creative destruction which reduces the cost of living. They also keep improving the relative appeal of their small share goods. While in the less innovation-intensive sector (snacks), large firms drive down the cost of living through improving their large share goods hence increase the dispersion of expenditure shares and drive down consumer valuation bias.

Admittedly, data from Nielsen might not be ideal to measure the economy-wide quality adjusted price since most of innovative items in the consumer basket may not be sold in grocery stores. Transaction-level data from other sources, such as NPD and online retailers can be utilized to perform further analysis.

The UPI framework is a tractable framework based on consumer demand theory to study the exact cost of living. The availability of transaction-level big data made it empirically possible to construct micro-macro consistent measure of price indices and to study product and firm dynamics. Its limitation lies in its sensitivity to small shares and the dependence on CES being appropriate in defining and measuring the consumer demand and the elasticity of substitution. On this front, there is ongoing work to compare this framework to other approaches such as hedonic approaches (Ehrlich et al. (2020)). In practice, a limitation is the common goods rule is ad hoc at this point. It would be interesting for future work to build models of dynamics of product entry and exit that underlies the common goods rule.

# Appendix A:

## A.1 Definition of the High-Tech Sector

We define high-tech based on the shares of workers in the STEM occupations of science, technology, engineering, and math, based on the methodology developed by Heckler (2005). High-tech sector in our analysis includes the following 14 four-digit NAICS industries, similar as Decker et al. (2018).

| **NAICS** | **Industry** |
|---|---|
| *Information and Communications Technology (ICT) High-Tech* | |
| 3341 | Computer and peripheral equipment manufacturing |
| 3342 | Communications equipment manufacturing |
| 3344 | Semiconductor and other electronic component manufacturing |
| 3345 | Navigational, measuring, electromedical, and control instruments manufacturing |
| 5112 | Software publishers |
| 5161 | Internet publishing and broadcasting |
| 5179 | Other telecommunications |
| 5181 | Internet service providers and Web search portals |
| 5182 | Data processing, hosting, and related services |
| 5415 | Computer systems design and related services |
| *Miscellaneous High-Tech* | |
| 3254 | Pharmaceutical and medicine manufacturing |
| 3364 | Aerospace product and parts manufacturing |
| 5413 | Architectural, engineering, and related services |
| 5417 | Scientific research-and-development services |

Table A.1: High-Technology Industries

## A.2 An Occupation-Based Measure of Demand for Skills

In this section, I present a demand for skills measure for the high-tech sector based on occupation using public available datasets.

The datasets used are 1980, 1990 and 2000 Decennial Census and ACS from 2005 to 2016 from IPUMS USA. We stack three consecutive years' ACS to obtain sample for the middle years (2006, 2009, 2012, 2015). The industry information in Census and ACS are not ideal. We manually construct a crosswalk from Census and ACS industry codes to NAICS2002 NAICS and keep the high-tech industries defined in Appendix A.1.

The advantage of Census and ACS is that they keep the occupation information for each individual. To define high and low skilled workers, I integrate the task score for each occupation from David and Dorn (2013) and define high-skilled workers as workers whose occupations have a higher-than-average abstract task score. Our measure is robust to other classification of high and low skilled workers.

Figure A.1 shows the relative price and quantity series for alternative definitions of skill (eduction and task). The skill premium flattened at 2000 and the flattening can be seen under both measures of skill.

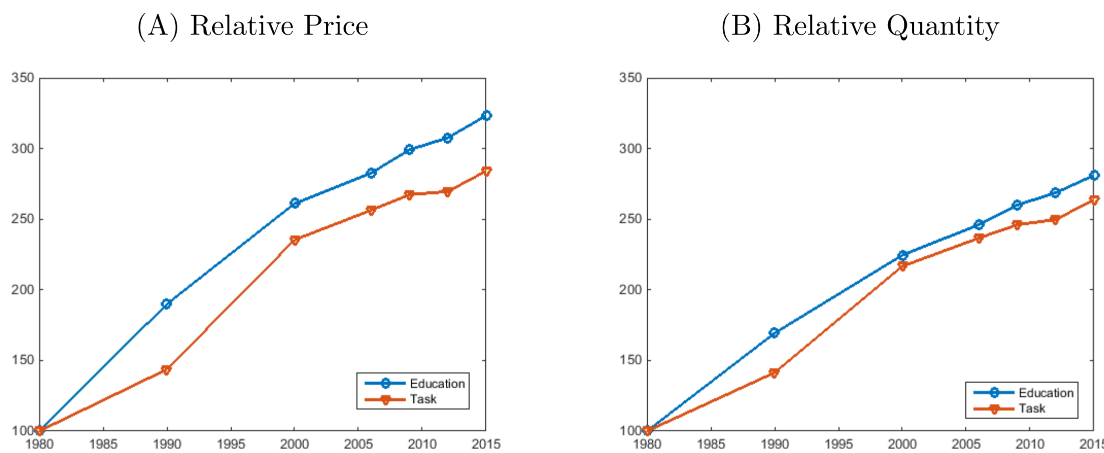


Figure A.1: This figure plots the relative price and quantity of skill under alternative definitions of skill. Panel (A) compares the relative price series. Panel (B) compares the relative quantity series.

## A.3 Final Good Producer's Problem

The optimization problem of the final good producer is

$$\max_{\{y_{jt}, Y_t\}} \left( P_t Y_t - \int_{j \in \Omega_t} p_{jt} y_{jt} dj \right) \tag{A.1}$$
$$\text{s.t.} \quad \ln Y_t = \frac{\int_{j \in \Omega_t} \ln y_{jt} dj}{M_t},$$

The Lagrangian function of the problem is

$$\mathcal{L} = P_t Y_t - \int_{j \in \Omega_t} p_{jt} y_{jt} dj - \lambda(\ln Y_t - \int_{j \in \Omega_t} \ln y_{jt} dj),$$

Solving this problem yields:

$$p_{jt} y_{jt} = \frac{P_t Y_t}{M_t}, \tag{A.2}$$

which further implies that

$$\ln P_t = \ln M_t + \frac{1}{M_t} \int_{j \in \Omega_t} \ln p_{jt} dj, \tag{A.3}$$

We choose the final good as the numeraire, i.e. $P_t = 1$.

## A.4 Analysis of the Bellman Equation

$$\tilde{v}(l^s_{a-1}) = \max_{\{l^s_a\}} \Big\{ \tilde{B}(l^s_a) - \tilde{w}^s(l^s_a + l_f) \\ -\frac{\varphi}{2}\tilde{w}^s(l^s_{a-1} + l_f)\Big[\frac{(l^s_a + l_f) - (1-\delta)(l^s_{a-1} + l_f)}{l^s_{a-1} + l_f}\Big]^2 + (1-\nu)\frac{1 + \lambda(l^s_a) - \mu}{1+r}\tilde{v}(l^s_a)\Big\}. \tag{A.4}$$

Redefine $\hat{r} = (r + \nu)/(1 - \nu)$

The FOC wrt $l^s_{a,t}$ gives:

$$\tilde{B}'(l^s_a) - \tilde{w}^s - \varphi\tilde{w}^s\Big[\frac{l^s_a + l_f}{l^s_{a-1} + l_f} - (1-\delta)\Big] + \frac{1 + \lambda(l^s_a) - \mu}{1+\hat{r}}\tilde{v}'(l^s_a) + \frac{\lambda'(l^s_a)}{1+\hat{r}}\tilde{v}(l^s_a) = 0 \tag{A.5}$$

The envelope condition gives:

$$\tilde{v}'(l^s_a) = \frac{\varphi w^s}{2}\Big[\frac{(l^s_{a+1} + l_f)^2}{(l^s_a + l_f)^2} - (1-\delta)^2\Big]. \tag{A.6}$$

Equation (A.5) and (A.6) give that along the balanced growth path,

$$\varphi\tilde{w}^s\Big[\frac{l_a^s+l_f}{l_{a-1}^s+l_f}-(1-\delta)\Big]=\tilde{B}'(\bar{l}_a^s)-\tilde{w}^s+ \\ \frac{1+\lambda(l_a^s)-\mu_t}{1+\hat{r}}\frac{\varphi\tilde{w}^s}{2}\Big[\frac{(l_{a+1}^s+l_f)^2}{(l_a^s+l_f)^2}-(1-\delta)^2\Big]+\frac{\lambda'(l_a^s)}{1+\hat{r}}\tilde{v}(l_a^s). \quad (A.7)$$

Let $\bar{a}$ be the age when $l_{\bar{a}}^s=\bar{l}$, then I can solve (A.7) backwardly as:

$$\varphi\tilde{w}^s\Big[\frac{l_{\bar{a}-j}^s+l_f}{l_{\bar{a}-j-1}^s+l_f}-(1-\delta)\Big]=\tilde{B}'(\bar{l}_{\bar{a}-j}^s)-\tilde{w}^s+ \\ \frac{1+\lambda(l_{\bar{a}-j}^s)-\mu_t}{1+\hat{r}}\frac{\varphi\tilde{w}^s}{2}\Big[\frac{(l_{\bar{a}+1-j}^s+l_f)^2}{(l_{\bar{a}-j}^s+l_f)^2}-(1-\delta)^2\Big]+\frac{\lambda'(l_{\bar{a}-j}^s)}{1+\hat{r}}\tilde{v}(l_{\bar{a}-j}^s). \quad (A.8)$$

$\bar{l}$ and $\tilde{v}(\bar{l})$ should satisfy:

$$\varphi\delta\tilde{w}^s=\tilde{B}'(\bar{l})-\tilde{w}^s+\frac{1+\lambda(\bar{l})-\mu}{1+\hat{r}}\frac{\varphi\tilde{w}^s}{2}(2\delta-\delta^2)+\frac{\lambda'(\bar{l})}{1+\hat{r}}\tilde{v}(\bar{l}), \quad (A.9)$$

$$\frac{\hat{r}+\mu-\lambda(\bar{l})}{1+\hat{r}}\tilde{v}(\bar{l})=\tilde{B}(\bar{l})-\tilde{w}^s(\bar{l}+l_f)(1+\frac{\varphi}{2}\delta^2). \quad (A.10)$$

## A.5 Existence and Uniqueness of $\bar{l}$

This section shows under certain conditions, there exist a unique value of $\bar{l}$ as the solution to (A.9). To begin with, note that $B(\bar{l})=B_0\bar{l}^{\eta_1}$, where $B_0=L^u\eta_0$.

Equation (A.9) can be written as:

$$\varphi w^s\delta - B'(\bar{l}) + w^s - \frac{1+\lambda(\bar{l})-\mu}{1+\hat{r}}\frac{\varphi w^s}{2}(2\delta-\delta^2) = \frac{\lambda'(\bar{l})}{r-\lambda(\bar{l})+\mu}\Big(B(\bar{l}) - w^s\bar{l} - \frac{\varphi}{2}w^s\bar{l}\delta^2\Big). \tag{A.11}$$

Redefine $\bar{l} = l^L - l_f$, then I have

$$\varphi w^s\delta - B'(\bar{l}) + w^s - \frac{1+\lambda(\bar{l})-\mu}{1+\hat{r}}\frac{\varphi w^s}{2}(2\delta-\delta^2) = \frac{\lambda'(\bar{l})}{r-\lambda(\bar{l})+\mu}\Big(B(\bar{l}) - w^s(\bar{l}+l_f) - \frac{\varphi}{2}w^s(\bar{l}+l_f)\delta^2\Big)$$

$\Longrightarrow$

$$\begin{aligned}
w^s(1+\varphi\delta) + \frac{\varphi}{2}(\delta^2-2\delta)\frac{1+\lambda_0\bar{l}^\theta-\mu}{1+\hat{r}} + \frac{\lambda_0\theta\bar{l}^{\theta-1}}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}w^2(\bar{l}+l_f) \\
+\frac{\varphi}{2}w^s\delta^2(\bar{l}+l_f)\frac{\lambda_0\theta\bar{l}^{\theta-1}}{\hat{r}+\mu-\lambda_0\bar{l}^\theta} \\
= \Big[\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\Big] \times B_0\bar{l}^{\eta_1-1}
\end{aligned}$$

$\Longrightarrow$

$$\begin{aligned}
\Big[\frac{1+\varphi\delta}{1+\frac{\varphi}{2}\delta^2} + \frac{\frac{\varphi}{2}(\delta^2-2\delta)}{1+\frac{\varphi}{2}\delta^2}\frac{1+\lambda_0\bar{l}^\theta-\mu}{1+\hat{r}} + \frac{\lambda_0\theta\bar{l}^{\theta-1}(\bar{l}+l_f)}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\Big]\Big[\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{\hat{r}+\mu-\lambda_0\bar{l}^\theta}\Big]^{-1} \\
= \frac{B_0\bar{l}^{\eta_1-1}}{\big(1+\frac{\varphi}{2}\delta^2\big)w^s}
\end{aligned} \tag{A.12}$$

LHS of equation (A.12) can be written as

$$LHS = 1 + \Big[1 - \eta_1 + C \cdot (\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}) + \frac{\lambda_0 \theta \bar{l}^{\theta-1} l_f}{\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}}\Big]\Big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}}\Big]^{-1}$$
$$= 1 + \Big[1 - \eta_1 + C \cdot (\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}) - \eta_1 \frac{l_f}{\bar{l}}\Big]\Big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}}\Big]^{-1} + \frac{l_f}{\bar{l}}$$
$$= 1 + \Big[1 - \eta_1 + C \cdot (\hat{r} + \mu - \lambda_0 \bar{l}^{\theta})\Big]\Big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}}\Big]^{-1} + \frac{l_f}{\bar{l}}\Big\{1 - \eta_1 \Big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{\hat{r} + \mu - \lambda_0 \bar{l}^{\theta}}\Big]^{-1}\Big\}$$

where

$$C = \frac{1}{1+\hat{r}} \frac{\frac{\varphi}{2}(2\delta - \delta^2)}{1 + \frac{\varphi}{2}\delta^2}.$$

Let

$$f_1(\bar{l}) = \frac{B_0 \bar{l}^{\eta_1 - 1}}{(1 + \frac{\varphi}{2}\delta^2) w^s} - \frac{l_f}{\bar{l}}\Big\{1 - \eta_1 \big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{r + \mu - \lambda_0 \bar{l}^{\theta}}\big]^{-1}\Big\}$$

$$f_2(\bar{l}) = \big[1 - \eta_1 + C(r + \mu - \lambda_0 \bar{l}^{\theta})\big]\big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{r + \mu - \lambda_0 \bar{l}^{\theta}}\big]^{-1} + 1$$

$f_1(\bar{l})\bar{l} = f_2(\bar{l})\bar{l}$ gives

$$\frac{B_0 \bar{l}^{\eta_1}}{(1 + \frac{\varphi}{2}\delta^2) w^s} - l_f \Big\{1 - \eta_1 \big[\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{r + \mu - \lambda_0 \bar{l}^{\theta}}\big]^{-1}\Big\} = \frac{(1 - \eta_1)\bar{l} + C\bar{l}(r + \mu - \lambda_0 \bar{l}^{\theta})}{\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{r + \mu - \lambda_0 \bar{l}^{\theta}}} + \bar{l}, \tag{A.13}$$

and

$$\frac{B_0 \bar{l}^{\eta_1}}{(1 + \frac{\varphi}{2}\delta^2) w^s} = \frac{(1 - \eta_1)\bar{l} + C\bar{l}(r + \mu - \lambda_0 \bar{l}^{\theta}) - l_f \eta_1}{\eta_1 + \frac{\lambda_0 \theta \bar{l}^{\theta}}{r + \mu - \lambda_0 \bar{l}^{\theta}}} + \bar{l} + l_f \tag{A.14}$$

Define the LHS of equation (A.14) to be new $f_1$ and RHS to be new $f_2$. We want to find conditions under which $df_1/d\bar{l} < df_2/d\bar{l}$

The LHS of the inequality can be written as

$$\frac{df_1}{d\bar{l}} = \frac{\eta_1}{\bar{l}} f_1 = \frac{\eta_1}{\bar{l}} f_2$$

. We now calculate the RHS of the inequality.

$$\begin{aligned}\frac{df_2}{d\bar{l}} &= 1 + \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - C\lambda_0\theta\bar{l}^\theta}{\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}} \\ &\quad - \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\left(\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}\right)^2} \times \frac{\lambda_0\theta^2(r+\mu)\bar{l}^{\theta-1}}{(r+\mu-\lambda_0\bar{l}^\theta)^2}\end{aligned}$$

Hence we need to have

$$\begin{aligned}&\frac{\eta_1}{\bar{l}} \times \Big[\frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}} + l_f\Big] \\ &< \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - C\lambda_0\theta\bar{l}^\theta}{\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}} + 1 - \eta_1 \\ &- \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\left(\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}\right)^2} \times \frac{\lambda_0\theta^2(r+\mu)\bar{l}^{\theta-1}}{(r+\mu-\lambda_0\bar{l}^\theta)^2}\end{aligned}$$

Multiply both sides by $\eta_1 + \frac{\lambda_0\theta\bar{l}}{r+\mu-\lambda_0\bar{l}^\theta}$ and organize the terms, we have

$$\begin{aligned}&\eta_1(1-\eta_1) + \eta_1 C(r+\mu-\lambda_0\bar{l}^\theta) + \eta_1\frac{l_f}{\bar{l}}\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta} \\ &< (1-\eta_1) + C(r+\mu-\lambda_0\bar{l}^\theta) - C\lambda_0\theta\bar{l}^\theta + (1-\eta_1) \times \left(\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}\right) \\ &- \frac{(1-\eta_1)\bar{l} + C\bar{l}(r+\mu-\lambda_0\bar{l}^\theta) - l_f\eta_1}{\eta_1 + \frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta}}\frac{\lambda_0\theta\bar{l}^\theta}{r+\mu-\lambda_0\bar{l}^\theta} \times \frac{r+\mu}{r+\mu-\lambda_0\bar{l}^\theta}\frac{1}{\bar{l}}\end{aligned}$$

## A.6 Proof of Lemma 2

$$\begin{aligned}\mu =&\Lambda_1 + \Lambda_1\lambda(l_1^s)(1-\nu) + \Lambda_1\lambda(l_2^s)[1+\lambda(l_1^s)-\mu](1-\nu)^2 + ...\\ &\Lambda_1\lambda(l_3^s)[1+\lambda(l_2^s)-\mu][1+\lambda(l_1^s)-\mu](1-\nu)^3 + ...\\ &\Lambda_1(1-\nu)^{\bar{a}-1}\Pi_{i=1}^{\bar{a}-2}[1+\lambda(l_i^s)-\mu]\lambda(l_{\bar{a}-1}^s) + ...\\ &\Lambda_1(1-\nu)^{\bar{a}}\Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s)-\mu]\frac{\lambda(l_{\bar{a}}^s)}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s)-\mu)}\end{aligned} \tag{A.15}$$

Define

$$\begin{aligned}M =& \Lambda_1 + \Lambda_1(1-\nu)[1+\lambda(l_1^s)-\mu] + \Lambda_1(1-\nu)^2[1+\lambda(l_1^s)-\mu][1+\lambda(l_2^s)-\mu]...\\ &+ \Lambda_1(1-\nu)^{\bar{a}-1}\Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s)-\mu] + \Lambda_1(1-\nu)^{\bar{a}}\Pi_{i=1}^{\bar{a}}[1+\lambda(l_i^s)-\mu]\frac{1}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s)-\mu)}\end{aligned}$$

Now let the $k$th term in equation (A.15) be combined with the $(k-1)$th term of $M(1-\mu)(1-\nu)$, we have

$$\begin{aligned}\mu + M(1-\mu)(1-\nu) =& \Lambda_1 + \Lambda_1(1-\nu)[1+\lambda(l_1^s)-\mu]...\\ &+ \Lambda_1(1-\nu)^2[1+\lambda(l_1^s)-\mu][1+\lambda(l_2^s)-\mu]...\\ &+ \Lambda_1(1-\nu)^{\bar{a}-1}\Pi_{i=1}^{\bar{a}-1}[1+\lambda(l_i^s)-\mu]...\\ &+ \Lambda_1(1-\nu)^{\bar{a}}\Pi_{i=1}^{\bar{a}}[1+\lambda(l_i^s)-\mu]\frac{1}{1-(1-\nu)(1+\lambda(l_{\bar{a}}^s)-\mu)}\\ =& M,\end{aligned}$$

So we can solve

$$M = \frac{\mu}{1-(1-\mu)(1-\nu)}. \tag{A.16}$$

## A.7 Algorithm of Solving the Balanced Growth Path

In this section, I describe the computation algorithm.

**Step 1:** We solve for $\bar{l}$ using equation (2.33) and calculate $v^s(\bar{l})$ using equation (2.32).

**Step 2:** Define the policy function $h(l)$ as the optimal choice of skilled labor per product line when the skilled labor per product line in the end of the last period is equal to $l$. We solve $v^s(l)$ and $h(l)$ using backward induction. Given $\hat{l}^s$, $h(\hat{l}_s)$, we can solve $l^s$, $h(l^s)$ and $v^s(l)$ such that $h(l^s) = \hat{l}^s$, using the following equations derived from the Bellman equation:

$$\begin{aligned}\varphi w^{s,s}\frac{(\hat{l}^s-(1-\delta)l^s)}{l^s} &= \frac{dB^s(\hat{l}^s)}{d\hat{l}^s} - w^{s,s} \\ &+ \frac{1+\lambda(\hat{l}^s)-\mu}{1+r}\frac{\varphi w^{s,s}}{2}\frac{h(\hat{l}^s)^2-(1-\delta)^2(\hat{l}^s)^2}{\hat{l^s}^2} + \frac{\lambda'(\hat{l}^s)}{1+r}v^s(\hat{l}^s)\end{aligned} \tag{A.17}$$

$$v^s(l^s) = B^s(\hat{l}^s) - w^{s,s}\hat{l}^s - \frac{\varphi w^{s,s}}{2}l^s\big[\frac{\hat{l}^s}{l^s} - 1 + \delta\big]^2 + \frac{1+\lambda(\hat{l}^s)-\mu^s}{1+r}v^s(\hat{l}^s) \tag{A.18}$$

In our current model, for a given optimal choice of $l$ in a certain period, there may not be a solution so that in the long run, the choices converge to $\bar{l}$, and may actually fluctuate around $\bar{l}$. We see our model as an approximation for a continuous time model. We will do backward induction starting from values close to $\bar{l}$ and get policy functions and value functions defined on a grid. Next, I interpolate firms' optimal choices when they do not fall exactly on the grids that are calculated from the backward induction. Specifically, we choose a very small $\epsilon_l > 0$, and let $h(\bar{l} - \epsilon_l) = \bar{l}$ and $v^s(\bar{l} - \epsilon_l) = v^s(\bar{l}) - \epsilon_l dv^s/d\bar{l}$.

**Step 3:** I determine $\mu^s$ as a function of $w^{s,s}$ and $\xi$, using the entry condition: (2.14):

$$\max_{l_0^{s,s}} \left\{ \lambda^E (l_0^{s,s} - \xi) \frac{v^s(l_0^{s,s})}{1+r} - w^{s,s} l_0^{s,s} \right\} = 0.$$

[1]

However, when the wage of the skilled worker $w^{s,s}$ is sufficiently high, $\mu^s$ may be lower than $\lambda(\bar{l})$, in which case the steady state with a positive entry does not exist.[2] In this case, the steady state will have no entry, $\mu = \lambda(\bar{l})$, and $\bar{l}$ satisfies the following equation:

$$\varphi w^{s,s}\delta - \frac{dB^s(\bar{l})}{d\bar{l}} + w^{s,s} - \frac{1}{1+r}\frac{\varphi w^{s,s}}{2}(2\delta - \delta^2) = \frac{\lambda'(\bar{l})}{r}\Big(B^s(\bar{l}) - w^{s,s}\bar{l} - \frac{\varphi}{2} w^{s,s}\bar{l}\delta^2\Big). \tag{A.19}$$

[1] The conditions in Proposition 1 require that $\mu^s$ should satisfy the following conditions:

$$\mu^s < (1-\eta_1)\frac{1+\frac{\varphi\delta_s^2}{2}}{\varphi\delta_s}(1+r) - r$$
$$\mu^s < \frac{(1-\eta_1)^2}{\theta}\frac{1+\frac{\varphi\delta_s^2}{2}}{\varphi\delta_s}(1+r) - r$$
$$\mu^s \geq \lambda_0\Big[\frac{1}{L^u\eta_0} w^{s,s}(1+\frac{\varphi}{2}(\delta)^2)\Big]^{\frac{\theta}{\eta_1 - 1}} - r$$

[2] This can be proved by noting that $\bar{l}$ and $v^s$ are both decreasing functions of $\mu$. If the term inside the maximization operator is negative, $\mu$ needs to decline to make the entry condition holds, and $\bar{l}$ will rise, making it possible to violate $\mu > \lambda(\bar{l})$.

**Step 4:** If there is a positive entry, $\Lambda_0^s$ is a function of $w^{s,s}$ and $\xi$. Otherwise, $\Lambda_0^s = 0$.

As we have obtained the optimal choices of firms $l_1^{s,s} = h(l_0^{s,s})$, $l_2^{s,s} = h(l_1^{s,s})$, $\cdots$, $l_{\bar{a}}^{s,s} = \bar{l} - \xi$, we can derive the distribution of firms as a function of $w^{s,s}$ and $\xi$.

Note that

$$\begin{cases} \Lambda_1^s = \Lambda_0^s \lambda^E(l_0^{s,s} - \xi) \\ \Lambda_a^s = \Lambda_1^s \Pi_{i=1}^{a-1}(1 + \lambda(l_i^{s,s}) - \mu^s), \quad \text{for } 2 \leq a \leq \bar{a}, \\ \Lambda_a^s = \Lambda_1^s \big(\Pi_{i=1}^{\bar{a}-1}(1 + \lambda(l_i^{s,s}) - \mu^s)\big)\big(1 + \lambda(l_{\bar{a}}^{s,s}) - \mu^s\big)^{a-\bar{a}}, \quad \text{for } a > \bar{a}. \end{cases}$$

We then calculate $\Lambda_0^s$ as a function of $\mu^s$ based on the following equation:

$$\begin{aligned} \mu^s &= M \times \{\Lambda_0^s \lambda^E(l_0^{s,s} - \xi) + \sum_{a=1}^{\infty} \lambda(l_a^{s,s}) \Lambda_a^s (1 - \nu)\} \\ &= M \times \Lambda_0^s \lambda^E(l_0^{s,s} - \xi) \times \Big\{1 + \lambda(l_1^s)(1 - \nu) + ... \\ &\sum_{a=2}^{\bar{a}-1} \lambda(l_a^s)(1 - \nu)^a \Pi_{i=1}^{a-1}(1 + \lambda(l_i^s) - \mu) + ... \\ &(1 - \nu)^{\bar{a}} \Pi_{i=1}^{\bar{a}-1}[1 + \lambda(l_i^s) - \mu] \frac{\lambda(l_{\bar{a}}^s)}{1 - (1 - \nu)(1 + \lambda(l_{\bar{a}}^s) - \mu)} \Big\} \end{aligned} \tag{A.20}$$

**Step 5:** If there is a positive entry, we solve $w^{s,s}$ from the market clearing condition of the skilled labor (2.26):

$$L^{s,s} = \Big[\frac{w^{s,s}}{\tau\chi}\Big]^{1/(\chi-1)} = \Lambda_0^s \Big\{ l_0^{s,s} + \lambda^E(l_0^{s,s} - \xi) l_1^{s,s} + \sum_{a=2}^{\bar{a}-1} \lambda^E(l_0^{s,s} - \xi)\big(\Pi_{i=1}^{a-1}(1 + \lambda(l_i^{s,s}) - \mu^s)\big) l_a^s + \lambda^E(l_0^{s,s} - \xi) \frac{\Pi_{i=1}^{\bar{a}-1}(1 + \lambda(l_i^{s,s}) - \mu^s)}{\mu^s - \lambda(\bar{l})} \bar{l} \Big\} \tag{A.21}$$

If there is no positive entry,

$$L^{s,s} = \Big[\frac{w^{s,s}}{\tau\chi}\Big]^{1/(\chi-1)} = \bar{l}. \tag{A.22}$$

**Step 6:** If there is positive entry, we determine $w^{u,s}$, using the market clearing condition:

$$w^{u,s} = \sum_{a=1}^{\infty} \frac{\Lambda_a^s}{q(l_a^{s,s})}.$$

If there is no positive entry,

$$w^{u,s} = \frac{1}{q(\bar{l})}.$$

## A.8 Entry Decision

Figure A.2 shows how the initial skilled labor is determined in the model: $l_0^s$ is chosen to maximize the entry value. The mass of entry adjust so that the entry value equals zero.

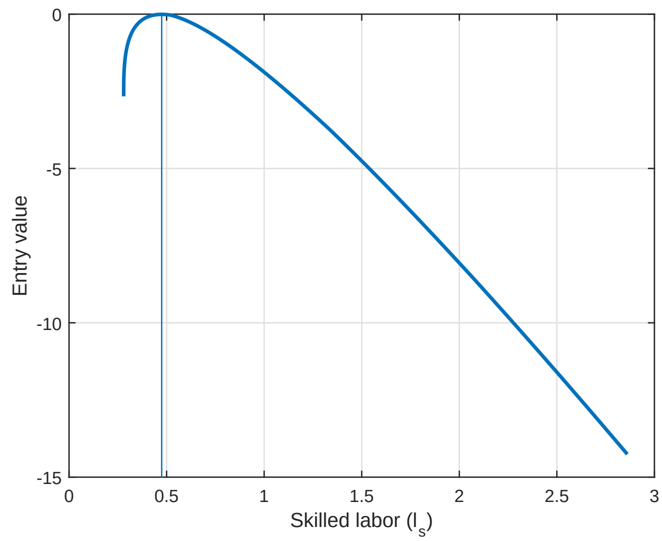


Figure A.2: Entry Decision

## A.9 Firm Size Distribution

Let $F(a, n)$ be the number of firms at age $a$ with $n$ product lines.

- When $a = 1$,

$$F(1,1) = \Lambda_0 \lambda^E (l_0^s - \xi). \tag{A.23}$$

- When $a > 1$,

$$F(a+1,n) = \sum_{k=[\frac{n+1}{2}]}^{2^{a-1}} F(a,k)(1-\nu) \sum_{l=0}^{k-[\frac{n+1}{2}]} C_{k-l}^{k} \mu^l (1-\mu)^{k-l} C_{n-(k-l)}^{k} \lambda^{n-(k-l)} (1-\lambda)^{2k-l-n}, \tag{A.24}$$

for $n \in [1, 2^a]$.

Hence the number of firms with $n$ product lines can be written as:

$$F^n = \sum_{a=1}^{\infty} F(a,n). \tag{A.25}$$

**Firm exit rate:**

The share of exiting firms between age $k$ and $l$ can be calculated as:

$$\sum_{a=k}^{l} \sum_{n=1}^{2^{a-1}} F(a,n)(1-\nu)\mu^n(1-\lambda)^n / \sum_{a=k}^{l} \sum_{n=1}^{2^{a-1}} F(a,n) + \nu. \tag{A.26}$$

# Bibliography


John M Abowd, Bryce E Stephens, Lars Vilhuber, Fredrik Andersson, Kevin L McKinney, Marc Roemer, and Simon Woodcock. The lehd infrastructure files and the creation of the quarterly workforce indicators. In *Producer dynamics: New evidence from micro data*, pages 149–230. University of Chicago Press, 2009.

Daron Acemoglu and David Autor. Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of labor economics*, volume 4, pages 1043–1171. Elsevier, 2011.

Daron Acemoglu, Ufuk Akcigit, Nicholas Bloom, and William R Kerr. Innovation, reallocation and growth. 2017.

Daron Acemoglu, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr. Innovation, reallocation, and growth. *American Economic Review*, 108(11):3450–91, 2018.

Philippe Aghion and Peter Howitt. A model of growth through creative destruction. *Econometrica*, 60:323–351, 1992.

Ufuk Akcigit and Sina T Ates. What happened to us business dynamism? Technical report, National Bureau of Economic Research, 2019.

Ufuk Akcigit and William R Kerr. Growth through heterogeneous innovations. *Journal of Political Economy*, 126(4):1374–1443, 2018.

David Argente, Salomé Baslandze, Sara Moreira, and Douglas Hanley. Patents to products: Innovation, product creation, and firm growth. Technical report, Working paper, 2019a.

David Argente, Chang-Tai Hsieh, and Munseob Lee. Measuring the cost of living in mexico and the us. 2019b.

Sina T Ates and Felipe Eduardo Saffie. Fewer but better: Sudden stops, firm entry, and financial selection. 2016.

David Autor et al. The polarization of job opportunities in the us labor market: Implications for employment and earnings. *Center for American Progress and The Hamilton Project*, 2010.

David H Autor and Brendan Price. The changing task composition of the us labor market: An update of autor, levy, and murnane (2003). *Unpublished Manuscript. Retrieved October*, 28:2015, 2013.

David H Autor, Lawrence F Katz, and Melissa S Kearney. Trends in us wage inequality: Revising the revisionists. *The Review of economics and statistics*, 90 (2):300–323, 2008.

Paul Beaudry, David A Green, and Benjamin M Sand. The great reversal in the demand for skill and cognitive tasks. *Journal of Labor Economics*, 34(S1):S199–S247, 2016.

C Lanier Benkard and Patrick Bajari. Hedonic price indexes with unobserved product characteristics, and application to personal computers. *Journal of Business & Economic Statistics*, 23(1):61–75, 2005.

Nicholas Bloom, Charles I Jones, John Van Reenen, and Michael Webb. Are ideas getting harder to find? Technical report, National Bureau of Economic Research, 2017.

Christian Broda and David E Weinstein. Globalization and the gains from variety. *The Quarterly journal of economics*, 121(2):541–585, 2006.

Christian Broda and David E Weinstein. Product creation and destruction: Evidence and price implications. *American Economic Review*, 100(3):691–723, 2010.

Ricardo J Caballero and Mohamad L Hammour. On the timing and efficiency of creative destruction. *The Quarterly Journal of Economics*, 111(3):805–852, 1996.

H David and David Dorn. The growth of low-skill service jobs and the polarization of the us labor market. *American Economic Review*, 103(5):1553–97, 2013.

Steven J Davis and John Haltiwanger. Labor market fluidity and economic performance. 2014.

Steven J Davis, R Jason Faberman, and John Haltiwanger. Labor market flows in the cross section and over time. *Journal of Monetary Economics*, 59(1):1–18, 2012.

Ryan Decker, John Haltiwanger, Ron Jarmin, and Javier Miranda. The role of entrepreneurship in us job creation and economic dynamism. *Journal of Economic Perspectives*, 28(3):3–24, 2014a.

Ryan Decker, John Haltiwanger, Ron Jarmin, and Javier Miranda. The secular decline in business dynamism in the us. *Unpublished draft, University of Maryland*, 2014b.

Ryan A Decker, John C Haltiwanger, Ron S Jarmin, and Javier Miranda. Changing business dynamism and productivity: Shocks vs. responsiveness. Technical report, National Bureau of Economic Research, 2018.

W Erwin Diewert. Hedonic regressions. a consumer theory approach. In *Scanner data and price indexes*, pages 317–348. University of Chicago Press, 2003.

W Erwin Diewert, Robert Feenstra, et al. Estimating the benefits and costs of new and disappearing products. *Vancouver School of Economics Discussion Paper*, pages 17–10, 2017.

Gabriel Ehrlich, John Haltiwanger, Ron Jarmin, David Johnson, and Matthew D Shapiro. Minding your ps and qs: Going from micro to macro in measuring prices and quantities. In *AEA Papers and Proceedings*, volume 109, pages 438–43, 2019a.

Gabriel Ehrlich, John Haltiwanger, Ron S Jarmin, David Johnson, and Matthew D Shapiro. Re-engineering key national economic indicators. *NBER Working Paper*, (w26116), 2019b.

Gabriel Ehrlich, John Haltiwanger, Ron Jarmin, Edward Olivares, Luke Pardue, Matthew Shapiro, and Laura Zhao. On the measurement of quality in price indices: Hedonic vs. upi approaches. *Working paper*, 2020.

Tim Erickson and Ariel Pakes. An experimental component index for the cpi: from annual computer data to monthly data on other goods. *American Economic Review*, 101(5):1707–38, 2011.

Robert C Feenstra. New product varieties and the measurement of international prices. *The American Economic Review*, pages 157–177, 1994.

Robert C Feenstra. Exact hedonic price indexes. *Review of Economics and Statistics*, 4(77):634–653, 1995.

John G Fernald and Charles I Jones. The future of us economic growth. *American economic review*, 104(5):44–49, 2014.

Lucia Foster, John C Haltiwanger, and Cornell John Krizan. Aggregate productivity growth: lessons from microeconomic evidence. In *New developments in productivity analysis*, pages 303–372. University of Chicago Press, 2001.

Fabio Ghironi and Marc J Melits. International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics*, 120(3):865–915, 2005.

Gene M Grossman and Elhanan Helpman. Quality ladders in the theory of growth. *The Review of Economic Studies*, 58(1):43–61, 1991.

John Haltiwanger, Ian Hathaway, and Javier Miranda. Declining business dynamism in the us high-technology sector. 2014a.

John Haltiwanger, Ian Hathaway, and Javier Miranda. Declining business dynamism in the us high-technology sector. 2014b.

John Haltiwanger, Henry R Hyatt, Erika McEntarfer, Liliana Sousa, and Stephen Tibbets. Firm age and size in the longitudinal employer-household dynamics data. *US Census Bureau Center for Economic Studies Paper No. CES-WP-14-16*, 2014c.

Daniel E Heckler. High-technology employment: a naics-based update. *Monthly Lab. Rev.*, 128:57, 2005.

Hugo Hopenhayn and Richard Rogerson. Job turnover and policy evaluation: A general equilibrium analysis. *Journal of Political Economy*, pages 915–938, 1993.

Hugo Hopenhayn, Julian Neira, and Rish Singhania. From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share. Technical report, National Bureau of Economic Research, 2018.

Colin J Hottman, Stephen J Redding, and David E Weinstein. Quantifying the sources of firm heterogeneity. *The Quarterly Journal of Economics*, 131(3):1291–1364, 2016.

Chang-Tai Hsieh and Peter J Klenow. Misallocation and manufacturing tfp in china and india. *The Quarterly Journal of Economics*, 124(4):1403–1448, 2009.

Henry R Hyatt and James R Spletzer. The recent decline in employment dynamics. *IZA Journal of Labor Economics*, 2(1):1–21, 2013.

Ron S Jarmin and Javier Miranda. The longitudinal business database. *Available at SSRN 2128793*, 2002.

Fatih Karahan, Benjamin Pugsley, and Ayşegül Şahin. Demographic origins of the startup deficit. Technical report, National Bureau of Economic Research, 2019.

Lawrence F Katz and Kevin M Murphy. Changes in relative wages, 1963–1987: supply and demand factors. *The quarterly journal of economics*, 107(1):35–78, 1992.

Lawrence F Katz et al. Changes in the wage structure and earnings inequality. In *Handbook of labor economics*, volume 3, pages 1463–1555. Elsevier, 1999.

Tor Jakob Klette and Samuel Kortum. Innovating firms and aggregate innovation. *Journal of political economy*, 112(5):986–1018, 2004.

Rasmus Lentz and Dale T Mortensen. An empirical model of growth through product innovation. *Econometrica*, 76(6):1317–1373, 2008.

Wassily Leontief. Ein versuch zur statistischen analyse von angebot und nachfrage. *Weltwirtschaftliches Archiv*, pages 1–53, 1929.

Raven S Molloy, Christopher L Smith, Riccardo Trezzi, Abigail Wozniak, et al. Understanding declining fluidity in the us labor market. Technical report, Brookings Papers on Economic Activity, 2016.

Ariel Pakes. A reconsideration of hedonic price indexes with an application to pc's. *American Economic Review*, 93(5):1578–1596, 2003.

Stephen J Redding and David E Weinstein. Measuring aggregate price indices with taste shocks: Theory and evidence for ces preferences. *The Quarterly Journal of Economics*, 135(1):503–560, 2020.

Diego Restuccia and Richard Rogerson. Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic dynamics*, 11(4): 707–720, 2008.

Paul M Romer. Endogenous technological change. *Journal of political Economy*, 98 (5, Part 2):S71–S102, 1990.

Sergio Salgado. Technical change and entrepreneurship. Technical report, mimeo, University of Minnesota, 2019.

Kazuo Sato. The ideal log-change index number. *The Review of Economics and Statistics*, pages 223–228, 1976.

Joseph A Schumpeter. *Capitalism, socialism and democracy*. New York: Harper and Brothers, 1942.

Robert G Valletta. Recent flattening in the higher education wage premium: Polarization, skill downgrading, or both? In *Education, skills, and technical change: Implications for future US GDP growth*. University of Chicago Press, 2018.

Yrjö O Vartia. Ideal log-change index numbers. *scandinavian Journal of statistics*, pages 121–126, 1976.

Lars Vilhuber et al. Lehd infrastructure s2014 files in the fsrdc. Technical report, 2018.